C.5 Difference Equations Solutions

C.5.1 In-lab section

1. (a) $M^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $M^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $M^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, and $M^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$, so we
guess that
$$M^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}.$$

   (b) Putting our guess to the test:

```
M = [1 1; 0 1];
M^25
```

```
ans =
```

```
    1      25
    0       1
```

we see that Matlab's answer matches our guess.

   (c) The guess holds for $n = 0$. Suppose it holds for some fixed n. I.e., for that particular value of n,
$$M^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}.$$

   Multiplying by M we get
$$M^{n+1} = M^n M = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & n+1 \\ 0 & 1 \end{bmatrix}.$$

   Thus, the guess holds for $n + 1$. Therefore, it must hold for all integers $n \geq 0$.

2. (a) `>> b = [2; 3]`

```
b =
```

```
    2
    3
```

```
>> A = [1, 1; 0, 1]
```

```
A =
```

```
    1       1
    0       1
```

```
>> b*A
??? Error using ==> *
Inner matrix dimensions must agree.

>> A*b

ans =

      5
      3
```

The product bA is nonsensical because b has only one column and A has two rows.

(b) `>> b'*A`

```
ans =

      2      5
```

```
>> A*b'
??? Error using ==> *
Inner matrix dimensions must agree.
```

The product Ab^T is nonsensical because A has two columns but b^T has only one row. The pattern is that the number of columns of the first factor must match the number of rows of the second.

3. The following Matlab function calculates output of the specified system given any input sequence.

```
function y = boing(x, sigma, omega)
% BOING - Return the output of a system with
% an impulse response that is a sinusoid with
% frequency omega (in radians per second) with
% a decaying exponential envelope. Arguments:
%   x      - A vector representing the input sequence.
%   sigma  - A scalar determining the rate of decay
%            of the impulse response.
%   omega  - A scalar giving the frequency of oscillation
%            in cycles per sample.
A = sigma*[cos(omega), -sin(omega); sin(omega), cos(omega)];
b = [0; 1];
c = sigma*[-cos(omega); sin(omega)];
% Initial state is zero
s = [0; 0];
for i=1:length(x)
```
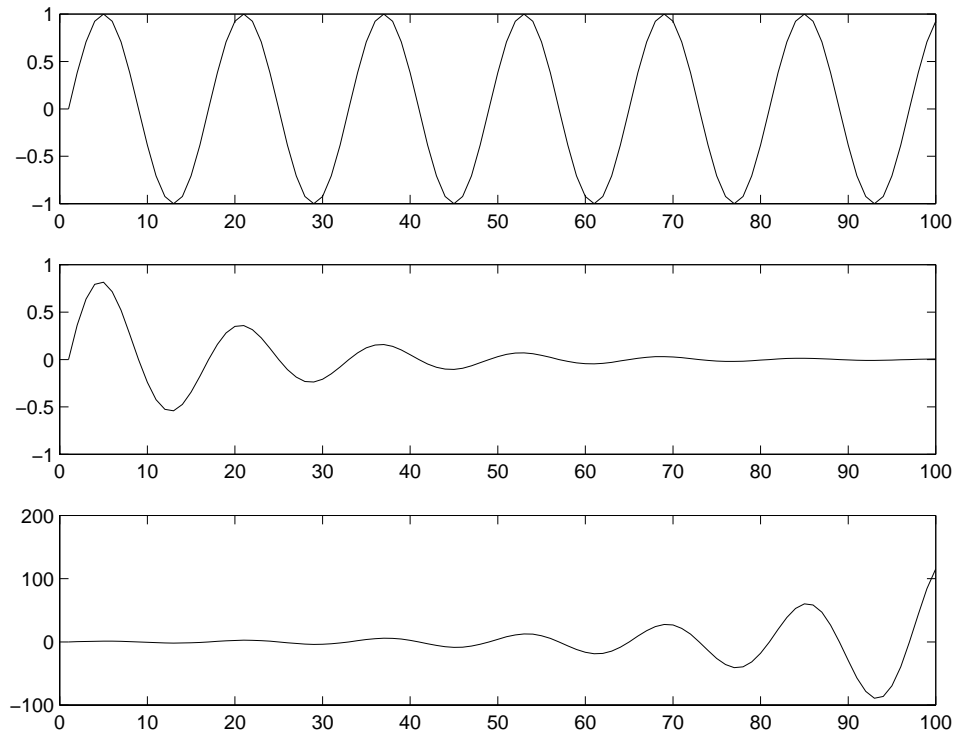
Figure C.10: Solutions to exercise 3 of the in-lab portion.

```
    y(i) = c'*s;
    s = A*s + b*x(i);
end
```

Notice how this code is written. The current state is stored in the variable `s`, which gets update *after* calculating the output inside the loop.

To use this to get the first 100 samples of the zero-state impulse response, we first construct the input sequence, and then calculate and plot it for each of the specified parameters:

```
x = [1, zeros(1,99)];
subplot(3,1,1); plot(boing(x, 1, pi/8));
subplot(3,1,2); plot(boing(x, 0.95, pi/8));
subplot(3,1,3); plot(boing(x, 1.05, pi/8));
```

The results are shown in figure C.10. The impulse response is a sinusoid when $\sigma = 1$, and a sinusoid multiplied by an exponential in the other two cases.

For part (d), we note that only the top result, where $\sigma = 1$, is periodic. The period is 16 samples.

For part (e), the system with $\sigma = 0.95$ is stable; the one with $\sigma = 1.05$ is unstable; and the one with $\sigma = 1.0$ is marginally stable. The amplitude of the unstable system output will grow

very rapidly until it eventually exceeds the numerical range representable by Matlab. It blows up.