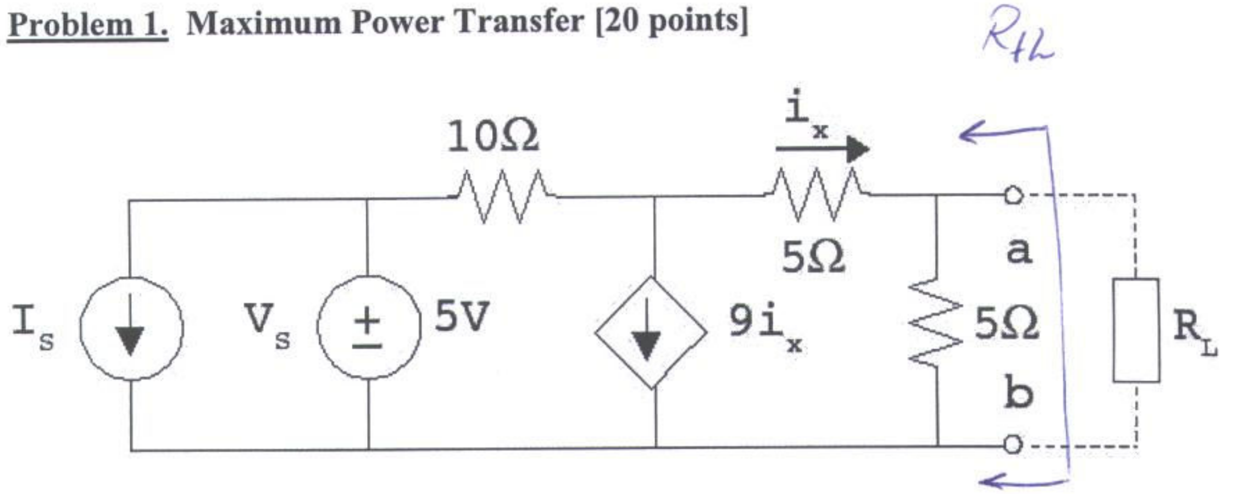
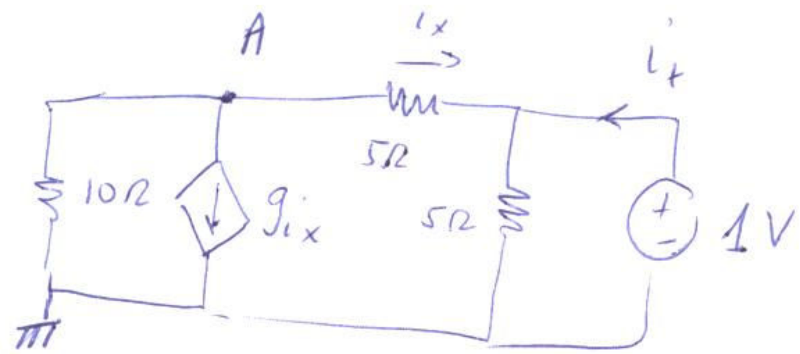
Name ______________________________

Problem 1. Maximum Power Transfer [20 points]



Find the optimal R_L such that the power delivered to R_L is maximized. What power is dissipated by R_L?

The power is maximized if $R_L = R_{th}$



KCL@A: $\frac{V_A}{10} + 9i_x + \frac{V_A - 1V}{5} = 0$

$i_x = \frac{V_A - 1V}{5} \Rightarrow \frac{V_A}{10} + 10\left(\frac{V_A - 1V}{5}\right) = 0$

$$V_A\left[\frac{1}{10} + 2\right] = 2V$$

$$V_A = \frac{2V}{2.1} = 0.95V$$

$$i_t = \frac{1V}{5} + \frac{1 - V_A}{5} = 0.21\ A$$

$\Rightarrow R_{th} = 4.76\ \Omega \Rightarrow R_L = 4.76\ \Omega$

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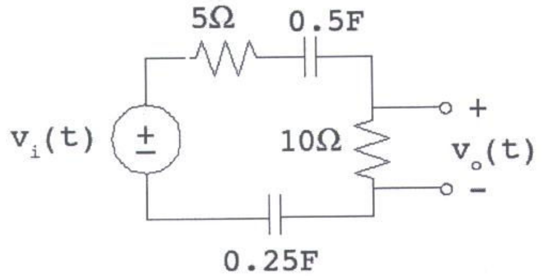
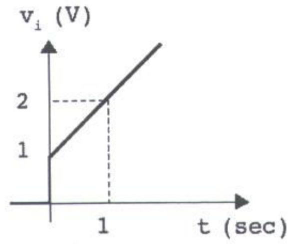
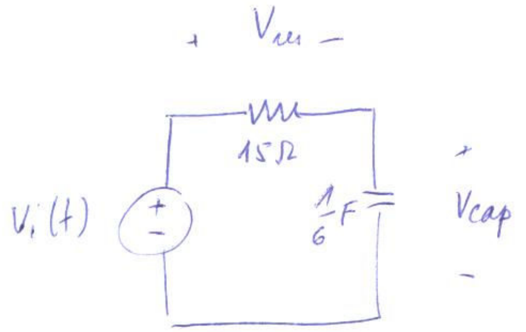
Problem 2. First-Order Transients [30 points]





Find and plot $v_o(t)$ for $t \in [0,2]$. (***Hint: It may be easier for you to first find i(t).)***
Annotate $v_o(0^+)$, $v_o(2)$.



$V_o = \frac{10}{15} V_{res} = \frac{2}{3} V_{res}$

$\tau = R_{eq} \cdot C_{eq} = 2.5s$

KVL: $V_{res} + V_{cap} = V_i$

$\Rightarrow R_{eq}\, i(t) + V_{cap}(t) = V_i(t)$

$R_{eq}\, i(t) + \frac{1}{C_{eq}} \int i(t)\,dt = v_i(t)$

$\tau \frac{di(t)}{dt} + i(t) = 1 \cdot C_{eq}$ for $t > 0$

$i(t) = i_c(t) + i_p(t)$. $i_p(t) = K$.

$\Rightarrow \tau \cdot \frac{di_p(t)}{dt} + i_p(t) = 1/6 \Rightarrow K = 1/6$

$i_c(t) = K' e^{-t/\tau}$

$\Rightarrow i(t) = \frac{1}{6} + K' e^{-t/\tau}$ and $i(0^+) = \frac{V_i(0^+)}{R_{eq}} = \frac{1}{15}$

$\Rightarrow 1/6 + K' = 1/15$ $K' = -\frac{1}{10}$

$\Rightarrow i(t) = \frac{1}{6} - \frac{1}{10} e^{-t/\tau} \Rightarrow V_o = 10\, i(t) = \underline{\underline{\frac{5}{3} - e^{-t/\tau}}}$

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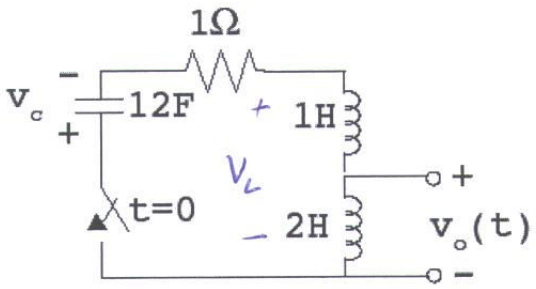
<u>Problem 3.</u> Second-Order Transients [30 points]



The switch has been open for a long time with the capacitor charged to $v_c = 10$ V. The switch closes at time $t = 0$. Find $v_o(t)$ for $t > 0$.

$$\frac{V_o}{V_L} = \frac{2}{3}$$

KVL: $V_{cap} + V_L + V_R = 0$

$$\frac{1}{C}\int i(t)\,dt + L\frac{di(t)}{dt} + \frac{i}{R} = 0$$

$$\Rightarrow \frac{1}{LC}\,i(t) + \frac{di(t)}{dt}\cdot\frac{1}{RL} + \frac{d^2 i(t)}{dt^2} = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{6}\ \text{rad/s} \qquad \zeta = \frac{\alpha}{\omega_0} = \frac{1}{2RL\cdot\omega_0} = 1$$

$\Rightarrow$ The solution is of the form: $K_1 + K_2 e^{-\omega_0 t} + K_3 t e^{-\omega_0 t}$

Finding K_1, K_2, K_3 from initial conditions:

$$\left.\begin{array}{l} i(0^+) = i(0^-) = 0 \Rightarrow K_1 + K_2 = 0 \\ i(\infty) = 0 \Rightarrow K_1 = 0 \end{array}\right\} \Rightarrow K_2 = 0$$

$$V_L(0^+) = L\frac{di(0^+)}{dt} = -10V \Rightarrow K_3 = -10/3$$

$$\Rightarrow i(t) = -10/3\, t e^{-\omega_0 t}$$

$$\Rightarrow V_o(t) = L\frac{di(t)}{dt} = \frac{-20}{3}\left[e^{-\omega_0 t} - \omega_0 t e^{-\omega_0 t}\right]$$

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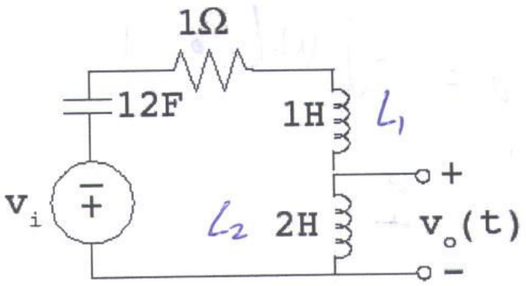
Problem 4. **Steady-State Sinusoidal Analysis [20 points]**



$$V_o = \frac{Z_{L_2}}{Z_{eq}} V_i(t) \quad (6)$$

$v_i(t) = 5 \cos (0.1\ t + 30°)$. What is $v_o(t)$?

$$\frac{V_o}{V_i} = \frac{(1)\ j\omega L_2\ (2)}{\underset{(3)}{R} + \underset{(4)}{\frac{1}{j\omega C}} + \underset{(5)}{j\omega(L_1+L_2)}} = \frac{\omega^2 L_2 C}{1-\omega^2(L_1+L_2)C + j\omega RC}$$

$$\left.\frac{V_o}{V_i}(0.1) = \frac{(0.1)^2 \cdot 2 \cdot 12}{1-(0.1)^2(3)12 + j\,0.1 \cdot 1 \cdot 12}\right\} (7)$$

$$\left.= \frac{0.24}{0.64 + j\,\cancel{0.12}\ 1.2} = -0.88 \angle 148.06°\right\}$$ process of finding Phasor (8,9)

$$\Rightarrow V_o(t) = -0.88 \cos(0.1t + 148.06°)$$ (10) Final Answer

BONUS [2 points]: Sketch the bode plot of the transfer function from v_i to v_o.