EECS40 Summer 2006

University of California Berkeley
Midterm Exam # 2
August 8th, 2006
Instructor: Octavian Florescu

Name: SOLUTION ____________________, ____________________
Last First

Student ID: ____________________, Signature____________________

1. This is a closed book exam. However you may use one page of handwritten double sided notes.
2. Non programmable calculators are allowed only.
3. Make sure to write down your name on every page.

Problem	Mark
1	/
2	/
3	/
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Total	/100

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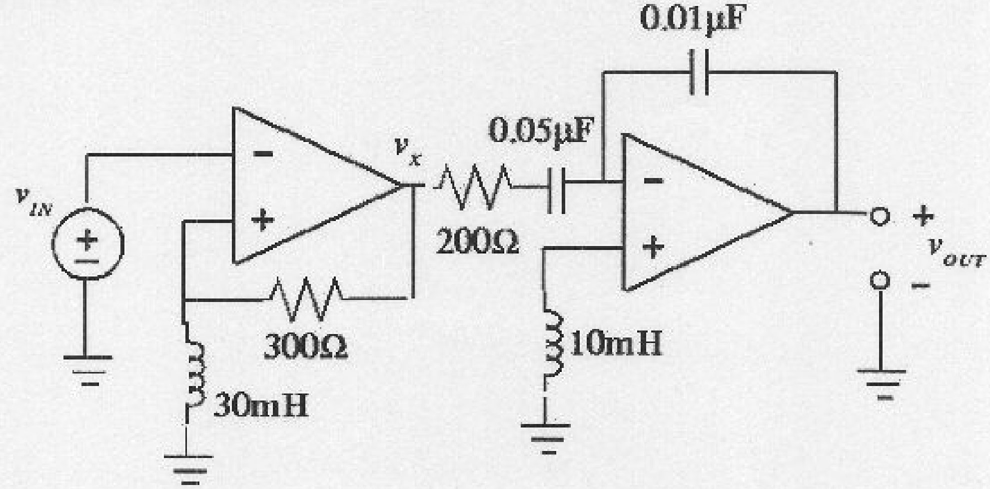
Problem 1. Op-Amps [25 points]



a. Find the transfer function v_X / v_{IN}.

We messed up! It's in positive feedback.
If you caught this: +5 pts.

Assuming negative feedback:

$$\text{KCL: } \frac{v_x - v_{in}}{300\Omega} = \frac{v_{in}}{j\omega\, 30mH} \rightarrow \frac{v_x}{v_{in}} = \frac{j\omega\, 0.03}{300 + j\omega\, 0.03} = \frac{1 + j\omega/10^4}{j\omega/10^4}$$

b. Find the transfer function v_{OUT} / v_X

The inductor does nothing because there is no current.

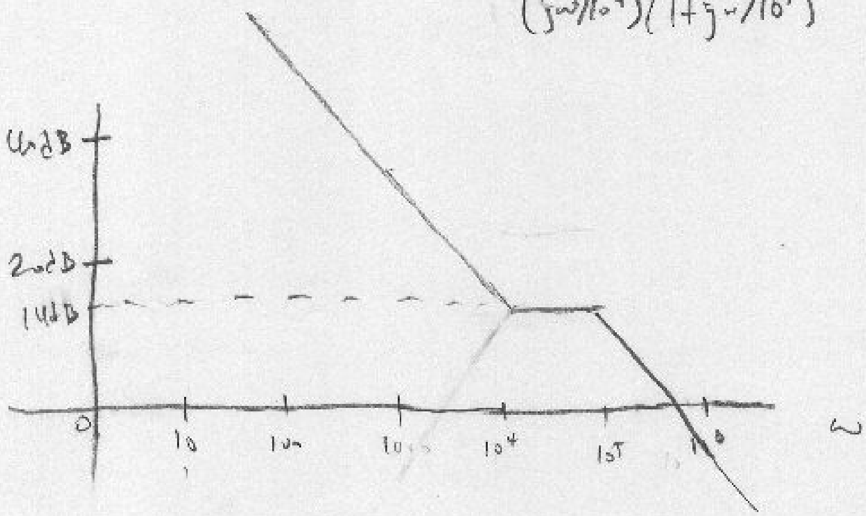
$$\text{KCL: } \frac{v_{out}}{1/j\omega 0.01\mu F} = \frac{-v_x}{200\Omega + 1/j\omega 0.05\mu F}$$

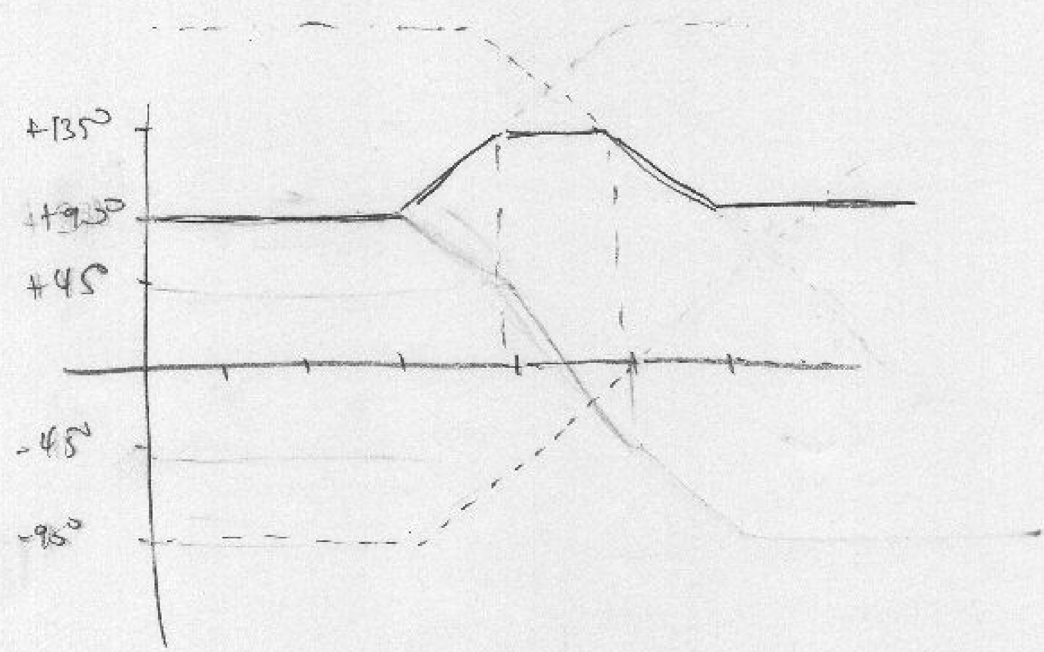
$$\rightarrow \frac{v_{out}}{v_x} = \frac{-1/j\omega\, 0.01\times10^{-6}}{200 + 1/j\omega\, 0.05\times10^{-6}} = \frac{-5}{1 + j\omega/10^5}$$

c. Find the transfer function v_{OUT}/v_{IN} and draw a bode plot.

$$\frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_x} \cdot \frac{v_x}{v_{in}} = \left(\frac{1+j\omega/10^4}{j\omega/10^4}\right)\left(\frac{-5}{1+j\omega/10^5}\right)$$

$$= -5\,\frac{(1+j\omega/10^4)}{(j\omega/10^4)(1+j\omega/10^5)}$$





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Problem 2. Semiconductors [25 points]

Consider the p-type silicon substrate below in figures 1 and 2. The *p* background concentration is $10^{15}\,cm^{-3}$. A *N*-region is implanted on top of the n background, with an acceptor concentration of $10^{17}\,cm^{-3}$.

Assume $n_i = 10^{10}\,cm^{-3}$

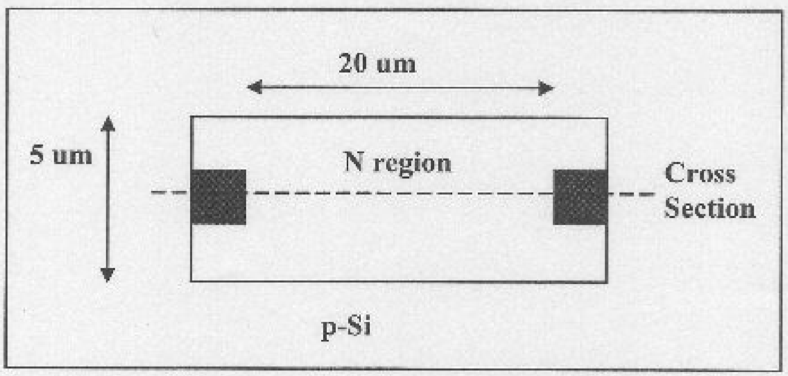


Figure 1: Top view of resistor

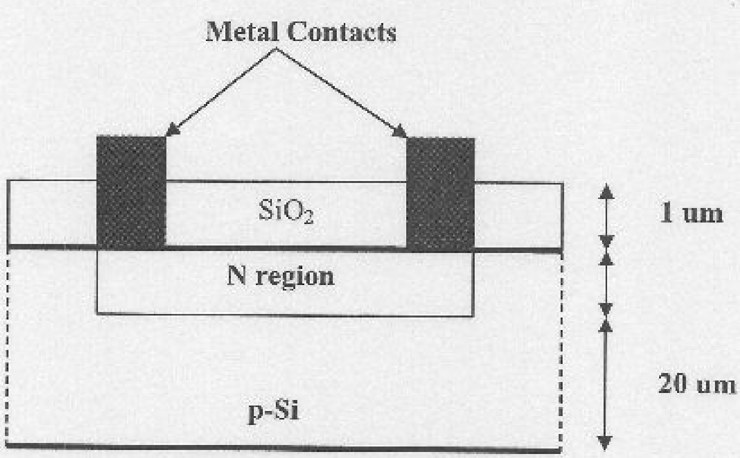


Figure 2: Cross section of resistor

a. Find the majority carrier concentration for the n and p regions.

$p: \; p_0 = N_a = 10^5$

$n: \; n_0 = N_d - N_a = 10^{17} - 10^5 = 9.90 \times 10^{16}$

b. Find the thickness of the depletion region. How much of that depletion region extends into the N region.

$\phi_0 = \frac{kT}{q} \ln\left(\frac{p_0 n_0}{n_i^2}\right) = 0.718V$ or. $\phi_0 = 60mV \log \frac{10^{17}}{10^{10}} \log \frac{10^{15}}{10^{10}} = 0.72V$

$$W_j = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{p_0} + \frac{1}{n_0}\right)\phi_0} = 96.9\,\mu m$$

$$X_{n_0} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{n_0}\right)\left(\frac{p_0}{n_0 + p_0}\right)} = 0.00969\,\mu m$$

c. Find the resistance of this device considering the depletion region effects.

$$R = \frac{1}{q n_0 \mu t} = \frac{1}{(1.6 \times 10^{-19})(9.9 \times 10^{16})(1000\ cm^2/Vs)(10 \times 10^{-4} cm - 9.69 \times 10^{-7} cm)}$$

$$= 63.2\ \Omega$$

d. What is the total surface area of the *pn* junction? Using this result, find the junction capacitance from the resistor to the substrate. Assume 0 bias conditions.

$$A = 20 \times 5 + 2(20 \times 10) + 2(5 \times 10) = 600\ \mu m^2$$

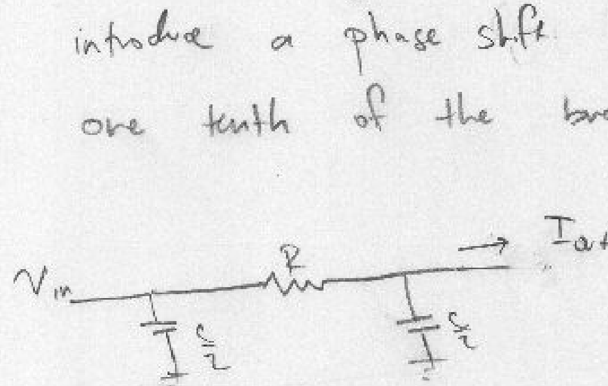
$$= 6 \times 10^{-6}\ cm^2$$

$$C = \frac{\varepsilon_s A}{W_j} = \frac{11.7 \times 8.85 \times 10^{-18}\ F/\mu m \quad 600\ \mu m^2}{96.9\ \mu m}$$

$$= 0.641\ pF$$

c. Assuming this capacitance can be lumped into two equal capacitances at each terminal of this device, what's the maximum frequency at which this device can operate like a resistor?

It stops acting like a resistor when we introduce a phase shift. This occurs at one tenth of the break frequency



$$I_{out} = \frac{V_{in}}{R \| Z_{ceq}} = \frac{V_{in}}{R \| \left(\frac{1}{j\omega C/2} + \frac{1}{j\omega C/2}\right)} = \frac{V_{in}}{R \| \frac{2}{j\omega C/2}}$$

$$= \frac{V_{in}}{R \| \frac{1}{j\omega C/4}} \rightarrow \frac{I_{out}}{V_{in}} = \frac{1}{R \| \frac{1}{j\omega C/4}} = \frac{1}{R} + j\omega \frac{C}{4}$$

$$= \frac{1}{R}\left(1 + j\omega \frac{RC}{4}\right) \rightarrow \omega_B = \frac{4}{RC} \rightarrow \omega_{MAX} = \frac{2}{5RC} = 9.87 \times 10^{12}\ rad/sec$$

$$= 1.57\ Hz$$

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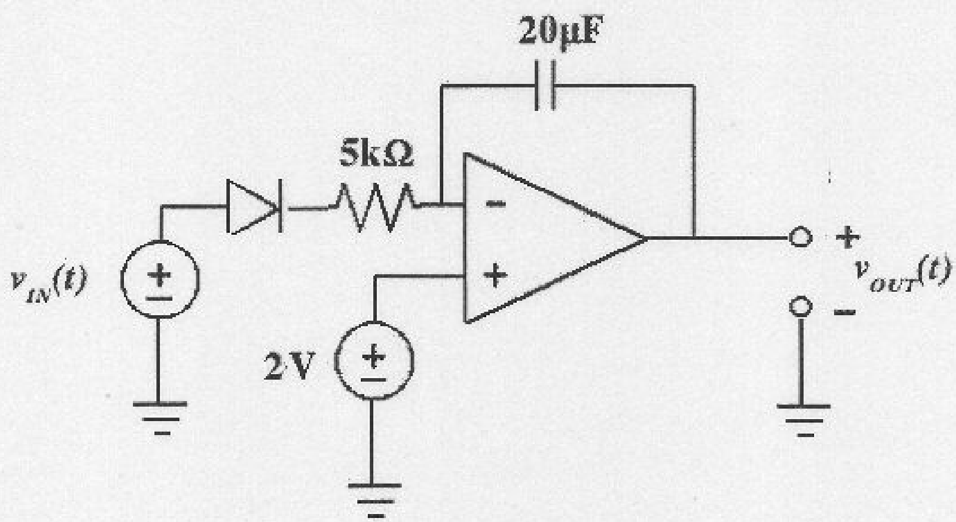
<u>Problem 3.</u> Diodes [20 points]



The diode has a built-in potential of 0.7 V. Use the large-signal model.

a. For what values of v_{IN} is the diode on?

$v_{in} \geq 2.7V$

b. What is the transfer function v_{OUT}/v_{IN} when the diode is on?

When finding the transfer function, we can ignore the 2V offset:

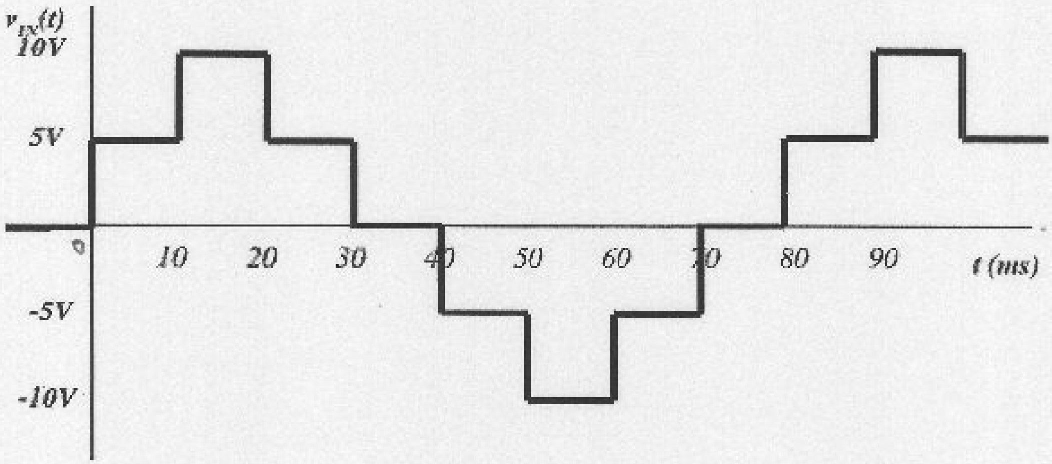
$\frac{V_{out}}{V_{in}} = \frac{1}{j\omega RC} = \frac{10}{j\omega}$ This is an integrator that scales by 10

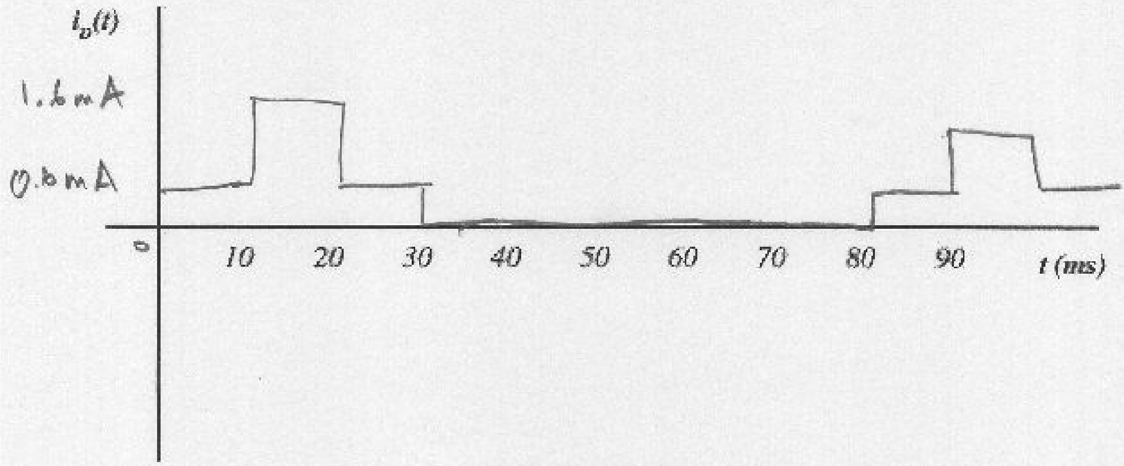
c. What is the transfer function v_{OUT}/v_{IN} when the diode is off?

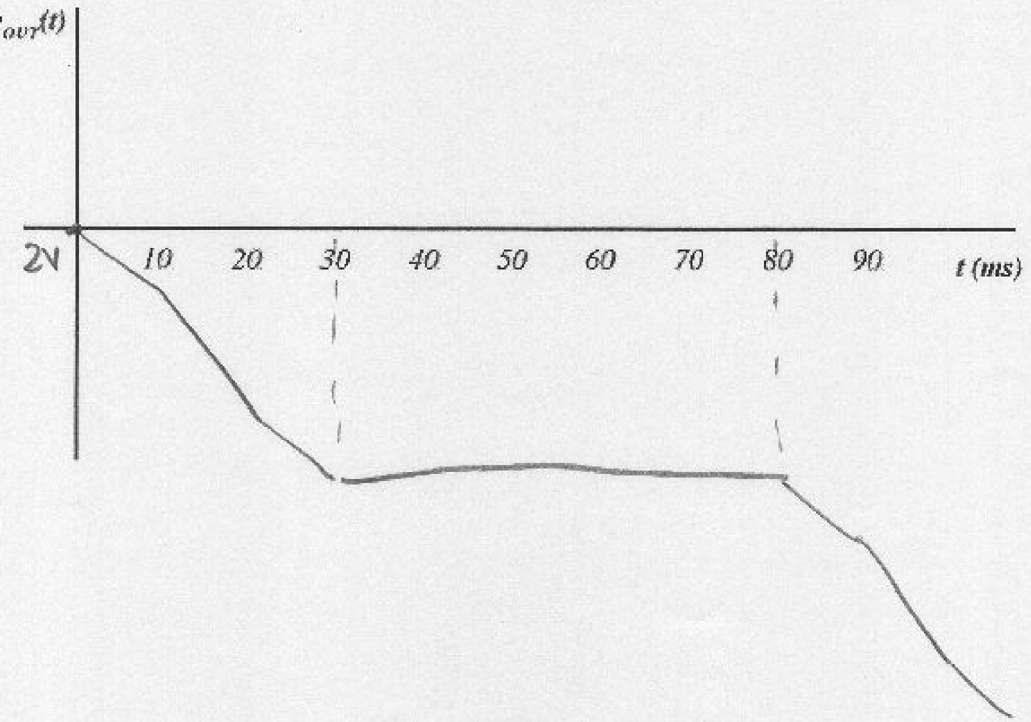
No current → V_{out} = const (cap holds voltage)

So $\frac{V_{out}}{V_{in}} = 0$

d. Plot $i_D(t)$ and $v_{OUT}(t)$ for the given $v_{IN}(t)$. Make sure to label your dependent axes with units.







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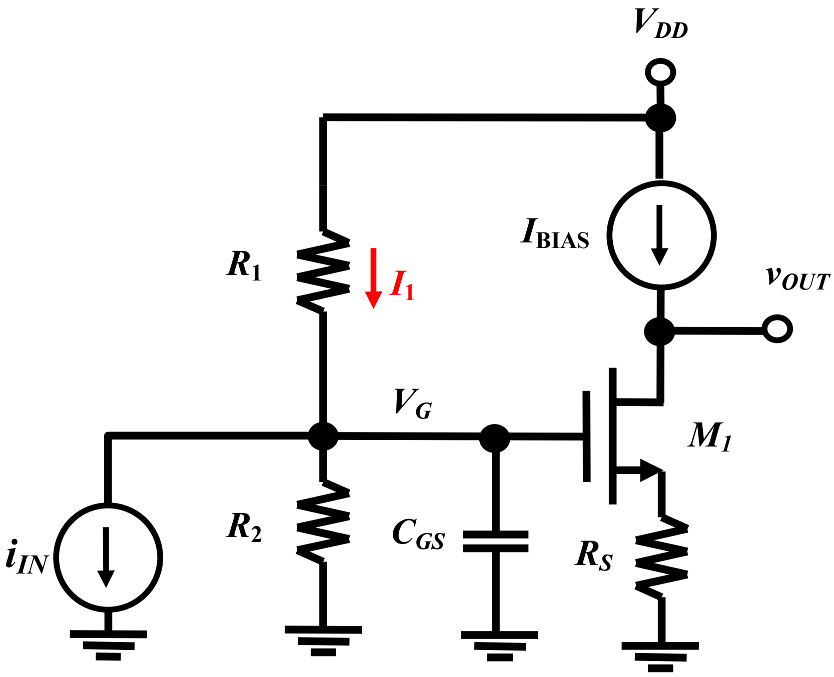
Problem 4. **MOSFET [30 points]**



Parameters:

$V_{DD} = 9V$
$I_{BIAS} = 2mA$

$R_1 = 3k\Omega$
$R_2 = 6k\Omega$
$R_S = 500\Omega$

$V_T(M_1) = 0.5V$
$\lambda(M_1) = 0.5V^{-1}$
$\mu_n C_{ox}(M_1) = 100\ uA/V^2$
$W/L(M_1) = 2$

a. The voltage $\boldsymbol{V_G}$ considering $\boldsymbol{I_{IN}}$ = 1mA. Note that $\boldsymbol{i_{IN} = i_{in} + I_{IN}}$ [5pts]

$$V_G = V_{DD} - I_1 \cdot R_1$$

$$I_1 = i_{IN} + \frac{V_G}{R_2}$$

$$V_G = 9V - (i_{IN} + \frac{V_G}{6k}) \cdot 3\text{k}$$

$$\frac{3}{2} V_G = 9V - i_{IN} \cdot 3\text{k}\Omega$$

$$\frac{3}{2} V_G = 9V - (i_{in} + 1mA) \cdot 3\text{k}\Omega$$

$$V_G = 4V - 2\text{k} \cdot i_{in}$$

V_G = 4V if i_{in} is 0 for DC analysis

Mistake 1: Ignoring I_{IN} = 1mA, and just do a voltage divide. (-0.5 point)
Mistake 2: Plug in the wrong values for unknown reasons. (-0.5 point)
Mistake 3: Wrong sign when setting up the equation(+ or -) (-0.5 point)

b. Find the source voltage of $\boldsymbol{M_1}$. [5pts]

$$V_S = I_{BIAS} \cdot R_S = 1V$$

c. Find $\boldsymbol{V_{DS}}$ using the values from **a.** and **b.** What region is M1 operating in? [5pts]

Assume M1 is in saturation region.

$$\text{I}_{\text{DS}} = \mu Cox \frac{W}{2L} (V_{GS} - Vt)^2 (1 + \lambda V_{DS})$$

$$2mA = (100\mu A / V^2) \frac{2}{2} (3V - 0.5V)^2 (1 + 0.5V^{-1} \cdot V_{DS})$$

Mistake 1:Wrong Vds (-0.5)
Mistake 2:No Vds but otherwise correct (-1)

$$V_{DS} = 4.4V, V_{GS} = 3V, Vt = 0.5V$$

$$V_{DS} > V_{GS} - Vt \quad \text{and} \quad V_{GS} > Vt$$

Assumption was correct, and M1 is in Saturation Region.

d. Find the transfer function $\boldsymbol{v_g / i_{in}}$. Note that i_{in} is a small signal without a DC component. [5pts]

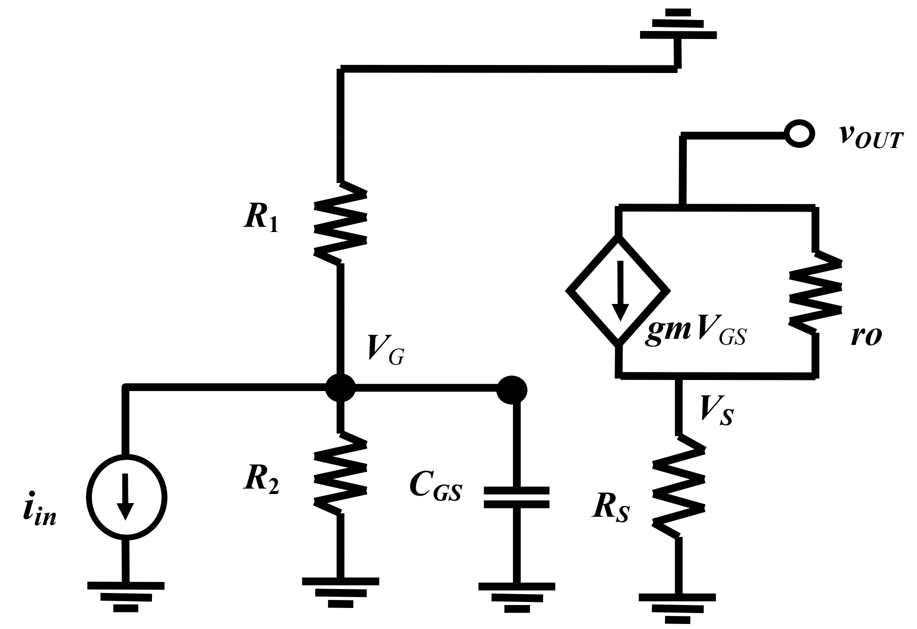
V_g = $-i_{in}$ (R_1//R_2// Z(C_{GS}))

$$\frac{V_g}{i_{in}} = -\frac{1}{(\frac{1}{R_1} + \frac{1}{R_2} + jwC_{GS})}$$

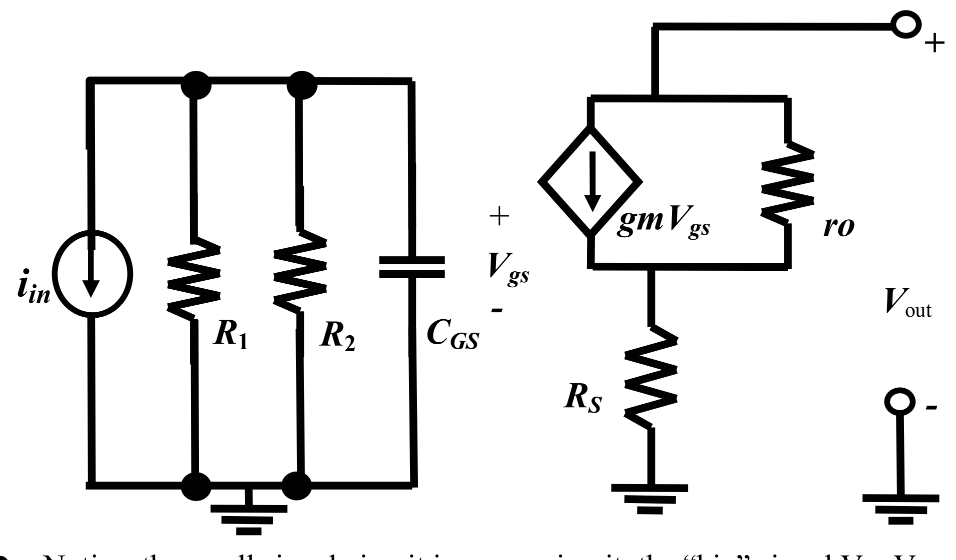
Common Mistake 1:Forgetting the negative sign (-0.5 point)
Common Mistake 2: Inverting the answer Vg/iin because the student forgot how to calculate impedance in parallel (-0.5 point)
Common Mistake 3: Both Mistake 1 and Mistake 2. (-1 point)

e. Sketch the small signal model of this configuration and find the transfer function v_{out}/i_{in}. [10 pts: 5pts for the small signal model, 5 pts for the calculations]

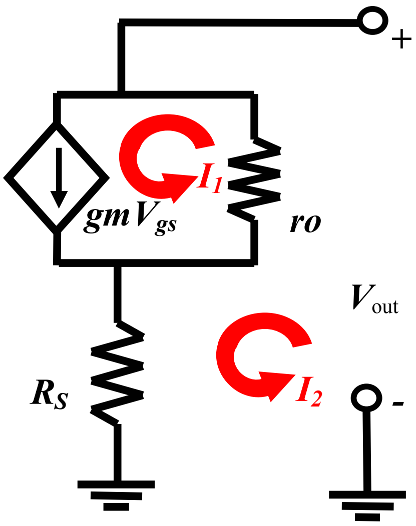
Step 1. Replace the circuit with all small circuit elements. That is eliminate all the DC (constant voltage or constant current) elements.



Step 2. which gives a small signal model of:



- Notice, the small signal circuit is a new circuit, the "big" signal V_G, V_S, and V_{OUT} are not here. Only small signal elements exist in a small signal model.
- C_{GS} is a misnomer. It is not the capacitor between the gate and source of M1.



$$\begin{cases} I_1 = gmVgs \\ I_1 = \dfrac{Vs - Vout}{ro} \\ Vout = (I_2 - I_1)ro + I_2 \cdot R_S \\ I_2 = \dfrac{Vs}{Rs} \end{cases}$$

$$gm = \mu_n C_{ox}(\frac{W}{L})(V_{GSQ} - V_t) = 0.5m$$

$$ro = \frac{1}{\lambda I_{DS}} = 1k$$

$$\begin{cases} I_1 = 0.5m\,(Vg - Vs) \\ I_1 = \dfrac{Vs - Vout}{1k} \\ vout = (I_2 - I_1)\cdot 1k + I_2 \cdot 500 \\ I_2 = \dfrac{Vs}{500} \end{cases}$$

Vout = - 1.375 Vg

$$\frac{Vout}{i_{in}} = \quad 1.375\,(R_1//R_2//\,Z(C_{GS}))$$

Common Mistake 1: The circuit said v_{OUT}, but this question asked for vout/vin. So the vout is supposed to be small signal output only, without the DC bias.

Common Mistake 2: Wrong small signal circuit. Some assumed C_{GS} is infinite and shorted C_{GS} in the small signal circuit.

BONUS [2 points]: Find the output impedance of this configuration.

$$Rout = \frac{1}{gm} // ro + Rs$$

$$Rout = ro + Rs + gm \cdot Rs \cdot ro$$