Problem 1

In order to determine whether r_{in} and r_{out} are "small" or "large," the following values will be used for the small-signal parameters of the transistors

$$g_m = \frac{2 \cdot I_D}{V_{DSAT}} = \frac{2 \cdot (250\mu A)}{(250mV)} = 2mS$$
$$g_{mb} = 0.3 \cdot g_m = 600\mu S$$
$$r_o = \frac{1}{\lambda \cdot I_D} = \frac{1}{(0.01)\cdot(250\mu A)} = 400k\Omega$$

Also, in calculating analytical expressions for r_{in} and r_{out}, it will be assumed that the input source has a source resistance of R_S and a load resistance R_L has been applied.

(a)

$$r_{in} = \infty$$
$$r_{out} = \left(\frac{1}{g_{m2} + g_{mb2}}\right) \ // \ r_{o1} = 384\Omega \ // \ 400k\Omega \approx 384\Omega$$

r_{in} is high; thus, voltage in

r_{out} is low; thus, voltage out

Hence, amplifier is best characterized using the voltage 2-port model

$$A_v = -g_{m1} \cdot \left(\frac{1}{g_{m2} + g_{mb2}} \ // \ r_{o1}\right) \approx \frac{-g_{m1}}{g_{m2} + g_{mb2}}$$

(b)

$$r_{in} = \frac{r_{o1} + \left(\frac{1}{g_{m2} + g_{mb2}} \ // \ R_L\right)}{(g_{m1} + g_{mb1}) \cdot r_{o1}} \approx \frac{r_{o1}}{(g_{m1} + g_{mb1}) \cdot r_{o1}} = \frac{1}{g_{m1} + g_{mb1}} = 384\Omega$$
$$r_{out} = \frac{1}{g_{m2} + g_{mb2}} \ // \ (r_{o1} + R_S + (g_{m1} + g_{mb1}) \cdot r_{o1} \cdot R_S) \approx \frac{1}{g_{m2} + g_{mb2}} = 384\Omega$$

r_{in} is low; thus, current in

r_{out} is low; thus, voltage out

Hence, amplifier is best characterized using the transresistance 2-port model

$$R_m = \frac{1}{g_{m2} + g_{mb2}}$$

(c)

$$r_{in} = \infty$$
$$r_{out} = (r_{o1} + R_X + (g_{m1} + g_{mb1}) \cdot r_{o1} \cdot R_X) \ // \ r_{o2}$$

The lowest that r_{out} can be in value is when $R_X = 0$. Hence, $r_{out} \geq \frac{r_o}{2} = 200k\Omega$

r_{in} is high; thus, voltage in

r_{out} is high; thus, current out

Hence, amplifier is best characterized using the transconductance 2-port model

$$G_m = \frac{g_{m1}}{1 + \frac{R_X}{r_{o1}} + (g_{m1} + g_{mb1}) \cdot R_X}$$

(d)

$r_{in} = \infty$

$$r_{out} = r_{o3} \ // \ (r_{o1} + \frac{1}{g_{m2} + g_{mb2}} + (g_{m1} + g_{mb1}) \cdot r_{o1} \cdot \frac{1}{g_{m2} + g_{mb2}}) \approx r_{o3} \ // \ (2 \cdot r_{o1}) = 267k\Omega$$

r_{in} is high; thus, voltage in

r_{out} is high; thus, current out

Hence, amplifier is best characterized using the transconductance 2-port model

One can use the transconductance formula derived in part (c) here, but noting that $R_X = \frac{1}{g_{m2} + g_{mb2}}$.

$$\therefore G_m = \frac{g_{m1}}{1 + \frac{1}{(g_{m2} + g_{mb2}) \cdot r_{o1}} + \frac{(g_{m1} + g_{mb1})}{(g_{m2} + g_{mb2})}} \approx \frac{g_{m1}}{1 + \frac{(g_{m1} + g_{mb1})}{(g_{m2} + g_{mb2})}} = \frac{g_{m1}}{2}$$

(e)

$$r_{in} = \frac{r_{o1} + (r_{o2} + r_{o3} + (g_{m2} + g_{mb2}) \cdot r_{o2} \cdot r_{o3}) \ // \ R_L}{(g_{m1} + g_{mb1}) \cdot r_{o1}}$$

The value for r_{in} depends heavily on R_L. Let's assume that $R_L << r_o$. Hence,

$$r_{in} \approx \frac{R_L}{(g_{m1} + g_{mb1}) \cdot r_{o1}}$$

$$r_{out} = (r_{o1} + R_S + (g_{m1} + g_{mb1}) \cdot r_{o1} \cdot R_S) \ // \ (r_{o2} + r_{o3} + (g_{m2} + g_{mb2}) \cdot r_{o2} \cdot r_{o3})$$

The lowest that r_{out} can be in value is when $R_S = 0$. Hence, $r_{out} \geq r_{o1} = 400k\Omega$

r_{in} is low; thus, current in

r_{out} is high; thus, current out

Hence, amplifier is best characterized using the current 2-port model

$A_i = -1$

(f)

$r_{in} = \infty$

$$r_{out} = \frac{1}{g_{m2}} \ // \ (r_{o1} + \frac{1}{g_{m3}} + (g_{m1} + g_{mb1}) \cdot r_{o1} \cdot \frac{1}{g_{m3}}) \approx \frac{1}{g_{m2}} = 500\Omega$$

r_{in} is high; thus, voltage in

r_{out} is low; thus, voltage out

Hence, amplifier is best characterized using the voltage 2-port model

$$A_v = \frac{-g_{m1}}{1 + \frac{1}{r_{o1} \cdot g_{m3}} + \frac{(g_{m1} + g_{mb1})}{g_{m3}}} \cdot r_{out} \approx \frac{-g_{m1}}{1 + \frac{1}{r_{o1} \cdot g_{m3}} + \frac{(g_{m1} + g_{mb1})}{g_{m3}}} \cdot \frac{1}{g_{m2}}$$

(g)

$r_{in} = \infty$

$$r_{out} = \frac{1}{g_{m3}} \ // \ \frac{r_{o1} + \frac{1}{g_{m2}}}{(g_{m1} + g_{mb1}) \cdot r_{o1}} \approx \frac{1}{g_{m3}} \ // \ \frac{1}{g_{m1} + g_{mb1}}$$

r_{in} is high; thus, voltage in

r_{out} is low; thus, voltage out

Hence, amplifier is best characterized using the voltage 2-port model

$$A_v = \frac{g_{m1}}{1 + \frac{1}{g_{m2} \cdot r_{o1}}} \cdot r_{out} \approx \frac{g_{m1}}{1 + \frac{1}{g_{m2} \cdot r_{o1}}} \cdot (\frac{1}{g_{m3}} \ // \ \frac{1}{g_{m1} + g_{mb1}})$$

(h)

$r_{in} = \infty$

$$r_{out} = r_{o3} \ // \ (r_{o1} + r_{o2} + (g_{m1} + g_{mb1}) \cdot r_{o1} \cdot r_{o2}) \approx r_{o3}$$

r_{in} is high; thus, voltage in

r_{out} is high; thus, current out

Hence, amplifier is best characterized using the transconductance 2-port model

One can use the transconductance formula derived in part (c) here, but noting that $R_X = r_{o2}$.

$$\therefore G_m = \frac{g_{m1}}{1 + \frac{r_{o2}}{r_{o1}} + (g_{m1} + g_{mb1}) \cdot r_{o2}}$$

(i)

$r_{in} = \infty$

$$r_{out} = r_{o2} \ // \ \frac{r_{o1} + r_{o3}}{(g_{m1} + g_{mb1}) \cdot r_{o1}} = r_{o2} \ // \ \frac{2 \cdot r_{o1}}{(g_{m1} + g_{mb1}) \cdot r_{o1}} \approx \frac{2}{g_{m1} + g_{mb1}}$$

r_{in} is high; thus, voltage in

r_{out} is low; thus, voltage out

Hence, amplifier is best characterized using the voltage 2-port model

$$A_v = \frac{g_{m1}}{1 + \frac{r_{o3}}{r_{o1}}} \cdot r_{out} \approx \frac{g_{m1}}{1 + \frac{r_{o3}}{r_{o1}}} \cdot \frac{2}{(g_{m1} + g_{mb1})}$$