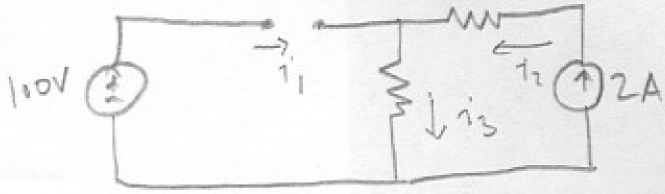
EE40 HW3 p1

P4.15

At steady state, the capacitors are like open circuits and the inductors like shorts:



Because of the open circuit, $\boxed{i_1 = 0A}$

Therefore all the current goes through the loop with i_2 and i_3

$\boxed{i_2 = i_3 = 2A}$

P4.36

(a) $i(0^+) = \dfrac{v_1(0^+) - v_2(0^+)}{R} = \dfrac{100V - 0V}{100k\Omega} = 1mA$

(b) $$\underbrace{i(t)R}_{v_R} + \underbrace{\frac{1}{C_2}\int_0^t i(t)\,dt + \overset{0}{v_2(0^+)}}_{v_2} + \underbrace{\frac{1}{C_1}\int_0^t i(t)\,dt - \overset{100}{v_1(0^+)}}_{-v_1} = 0$$

In differential form:

$$R\frac{di(t)}{dt} + \left(\frac{1}{C_1} + \frac{1}{C_2}\right) i(t) = 0$$

(c) $$\tau = \frac{R}{\left(\frac{1}{C_1}+\frac{1}{C_2}\right)} = \frac{RC_1C_2}{C_1+C_2} = \frac{(100k\Omega)(1\mu F)(1\mu F)}{(1\mu F + 1\mu F)} = \frac{10^5\,10^{-6}\,10^{-6}}{2\times10^{-6}}\,s$$

$= \frac{1}{20}s = \boxed{50ms}$

(d) $i(t) = Ke^{-t/\tau}$

$i(0^+) = 1mA \rightarrow K = 1ma$ so $\boxed{i(t) = (1mA)e^{-t/(50ms)}}$

(e) $v_2(t) = \dfrac{1}{C_2}\int_0^t i(t)\,dt + v_2(0^+)$

$$v_2(\infty) = \frac{1}{1\mu F}\int_0^\infty 1mA\, e^{-t/50ms}\,dt + 0$$

$$= 10^6\left[-50\times10^{-3}\times10^{-3}e^{-t/5\times10^{-3}}\right]_0^\infty = -50(0-1) = \boxed{50V}$$

EE40 HW3 p2

P 4.38

KVL: $i(t)R + L\frac{di(t)}{dt} = v(t)$

$2\frac{di(t)}{dt} + 10\,i(t) = 10t$

$i_p(t) = A + Bt$

$\frac{d}{dt}i_p(t) = B \quad \rightarrow \quad 2B + 10A + 10Bt = 10t$

$2B + 10A = 0 \quad 10B = 10 \quad \rightarrow B = 1,\ A = -0.2$

$i_p(t) = -0.2 + t$

$i_c(t) = Ke^{-t/(\frac{L}{R})} = Ke^{-5t}$

$i(t) = i_p(t) + i_c(t) = -0.2 + t + Ke^{-5t}$

$i(0) = 0 \rightarrow -0.2 + 0 + K = 0 \rightarrow K = 0.2$

$\boxed{i(t) = -0.2 + t + 0.2e^{-5t}}$

P4.45

KVL: $L\frac{di(t)}{dt} + i(t)R + v_c(t) = v_s \qquad i(t) = C\frac{dv_c(t)}{dt}$

$LC\frac{d^2}{dt^2}v_c(t) + RC\frac{d}{dt}v_c(t) + v_c(t) = v_s$

$\ddot{v}_c(t) + \frac{R}{L}\dot{v}_c(t) + \frac{1}{LC}v_c(t) = v_s/LC$

Since v_s is a constant, we will let $v_{cp}(t) = K$

$\therefore\ \dot{v}_{cp}(t) = 0$ and $\frac{1}{LC}v_{cp}(t) = v_s/LC \rightarrow v_{cp}(t) = v_s = 50V$

To find $v_{ce}(t)$, we look at the characteristic equation:

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ The roots are $s = \frac{R}{2L} \pm \frac{1}{2}\sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}$

$s_1 = -267.9$

$s_2 = -3732$

$v_c(t) = 50 + K_1e^{s_1t} + K_2e^{s_2t}$

$v_c(0) = 0 \rightarrow K_1 + K_2 = -50$

$i(0) = 0 \rightarrow \frac{dv_c(t)}{dt} = 0 \rightarrow s_1K_1 + s_2K_2 = 0$

$K_1 = -53.87 \quad K_2 = 3.867$

$\boxed{v_c(t) = 50 - 53.87e^{-267.9t} + 3.867e^{-3732t}}$

P4.51

KVL: $L\frac{di(t)}{dt} + i(t)R + \frac{1}{C}\int i(t)dt + i_c(0) = 10\cos(100t)$

$$L\frac{d^2}{dt^2}i(t) + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = -1000\sin(100t)$$

try $i_p(t) = A\sin 100t + B\cos 100t$

$\frac{d}{dt}i_p(t) = 100A\cos 100t - 100B\sin 100t$

$\frac{d^2}{dt^2}i_p(t) = -10000A\sin 100t - 10000B\cos 100t$ $\quad = -10^3\sin 100t$

$-10^4 A\sin 100t - 10^4 B\cos 100t + 5\times10^3 A\cos 100t - 5\times10^3 B\sin 100t + 10^4 A\sin 100t + 10^4 B\cos 100t$

$(-10^4 A - 5\times10^3 B + 10^4 A)\sin 100t + (-10^4 B + 5\times10^3 A + 10^4 B)\cos 100t = -10^3\sin 100t$

$-5\times10^3 B = -10^4 \qquad 5\times10^3 A = 0$

$B = 0.2 \qquad A = 0$

$i_p(t) = 0.2\cos 100t$

For the complementary solution:

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \qquad s = 25 \pm j96.82$

$i_c(t) = K_1 e^{-25t}\cos(96.82t) + K_2 e^{-25t}\sin(96.82t)$

$i(t) = 0.2\cos 100t + K_1 e^{-25t}\cos(96.82t) + K_2 e^{-25t}\sin(96.82t)$

$i(0^+) = 0 = 0.2 + K_1 \rightarrow K_1 = -0.2$

$\frac{di(0^+)}{dt} = 10 = -25K_1 + 96.82K_2 \rightarrow K_2 = 0.052$

$i(t) = 0.2\cos(100t) - 0.2e^{-25t}\cos(96.82t) + 0.052e^{-25t}\sin(96.82t)$

P5.32

$$Z = \frac{1}{j\omega C + \frac{1}{j\omega L}} = \frac{j\omega L}{1-\omega^2 LC} = \frac{j\omega\, 0.1}{1-\omega^2 10^{-6}}$$

for $\omega = 500 \quad Z = 66.67\,\Omega$

for $\omega = 1000 \quad Z = \frac{j100}{1-1} = \infty\,\Omega$

for $\omega = 2000 \quad Z = -j66.67\,\Omega$

EE40 HW3 p4

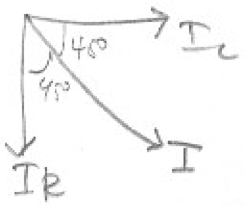
P 5.43

$$Z_{eq} = j\omega L + \frac{1}{\frac{1}{R} + j\omega C} = 100j + \frac{1}{10^{-2} + j10^{-2}} = 50 + j50 = 70.7\angle 45^\circ$$

$$I = \frac{100\cos 10^4 t}{70.7\angle 45^\circ} = \frac{100\angle 0}{70.7\angle 45} = 1.41\angle -45^\circ$$

$$I_R = \frac{1/j\omega C}{R + 1/j\omega C} I = \frac{-j100}{100 - j100} 1.41\angle -45^\circ = 1\angle -90^\circ$$

$$I_C = \frac{R}{R + 1/j\omega C} I = \frac{100}{100 - j100} 1.41\angle -45^\circ = 1\angle 0^\circ$$



P 5.44

KCL @ ①
$$\frac{V_1}{10} + \frac{V_1 - V_2}{5 + j15} = 1\angle 0^\circ$$

KCL @ ②
$$\frac{V_2}{-j10} + \frac{V_2 - V_1}{5 + j15} = 1\angle 30^\circ$$

$\xrightarrow{\text{solve}}$ $V_1 = 6.735\angle -38.54^\circ$

$V_2 = 16.25\angle -55.52^\circ$

P 5.46

KVL on ① $5I_1 + j15(I_1 - I_2) - 20 = 0$

KVL on ② $-j10 I_2 + (-10) + j15(I_2 - I_1) = 0$

Solving: $I_1 = 1.644\angle 80.54^\circ$

$I_2 = 2.977\angle 74.2^\circ$

P 5.47

Supermesh. $5I_1 + j5 I_2 + 10 I_2 - 10 = 0$

$I_1 - I_2 = 2$

Solving: $I_1 = 2$, $I_2 = 0$

EE40 HW 3 p5

P6.14

$$v_{in}(t) = V_M \cos \omega t = V_M \angle 0$$

$$v_{out}(t) = \int V_M \cos \omega t = -\frac{V_M}{\omega} \sin \omega t = \frac{V_M}{\omega} \cos(\omega t - 90^\circ) = \frac{V_M}{\omega} \angle -90^\circ$$

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{\omega} \angle -90^\circ = \frac{1}{j\omega}$$

P6.27

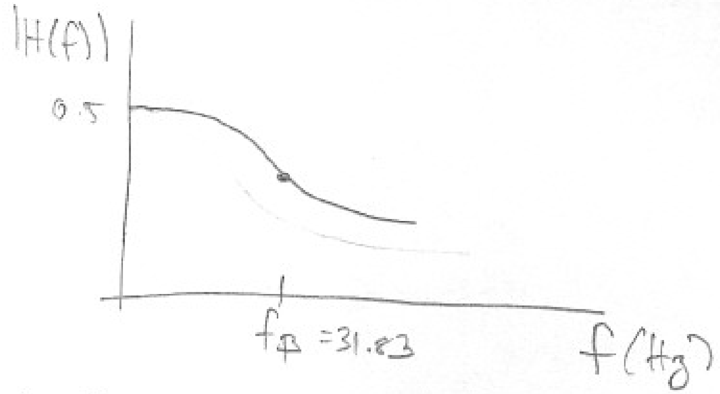
(a) Voltage Divider:

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2 + j\omega L} = \frac{R_2/(R_1+R_2)}{1 + j(\omega/\omega_B)}$$

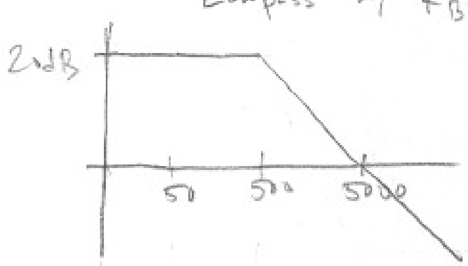
where $\omega_B = \frac{R_1+R_2}{L}$, $f_B = \frac{R_1+R_2}{2\pi L}$

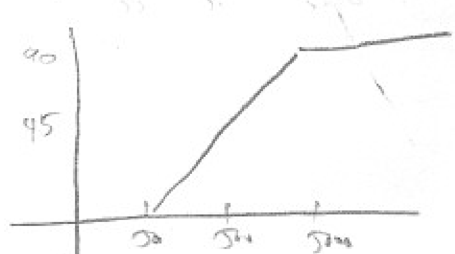
(b) $f_B = 31.83$ kHz

$H(0) = 0.5$



P6.46 Lowpass w/ $f_B = 500$





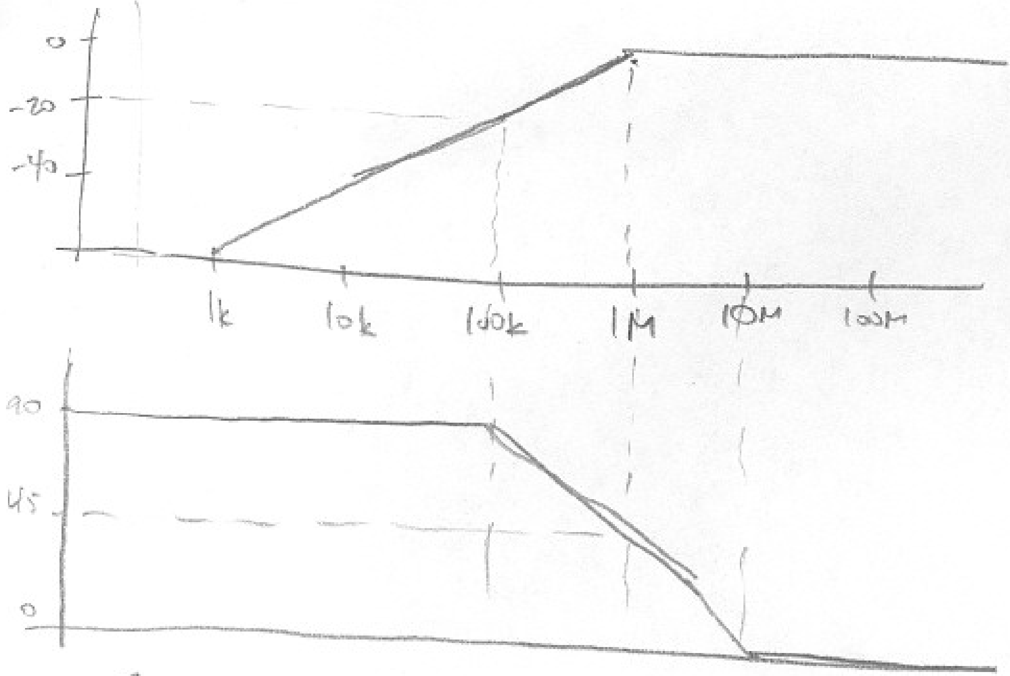
* The phase is reversed because of the "−" in the denominator

EE160 Hw 3 p6

P6.56 Voltage Divider:

$$H(\omega)=\frac{V_{out}}{V_{in}}=\frac{j\omega L}{R+j\omega L}=\frac{j\omega/(R/L)}{1+j\omega/(R/L)} \qquad f_B=\frac{R}{2\pi L}=1\,MHz$$

Highpass w/ f_B 1MHz



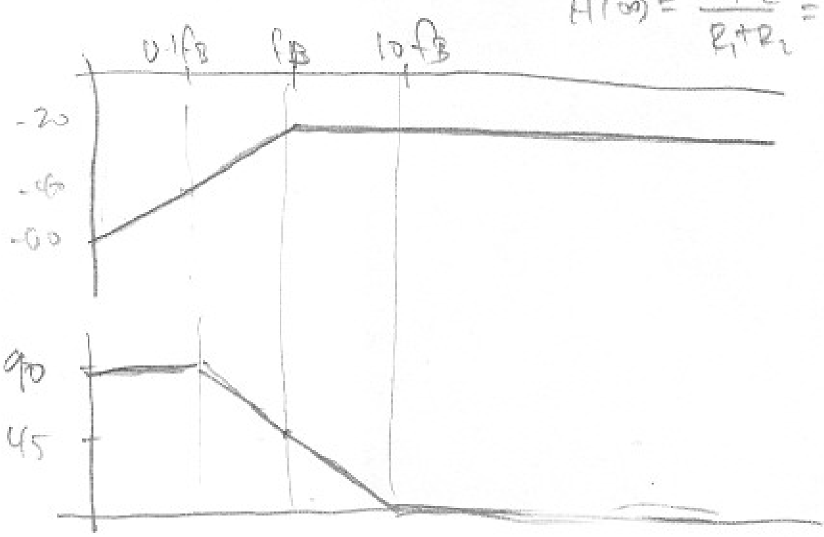
P6.57

Voltage Divider:

$$H(\omega)=\frac{V_{out}}{V_{in}}=\frac{R_2}{R_1+R_2+\frac{1}{j\omega C}}=\frac{R_2/(R_1+R_2)}{1+1/j\omega(R_1+R_2)C}=\frac{R_2}{R_1+R_2}\,\frac{j\omega/\omega_B}{1+j\omega/\omega_B}$$

HPF w/ $f_B=\frac{1}{2\pi C(R_1+R_2)}=15.92\,Hz$ $\qquad \omega_B=\frac{1}{C(R_1+R_2)}$

$$H(\infty)=\frac{R_2}{R_1+R_2}=0.1=-20dB$$



EE40 HW3 p7

Prob 1

(a) KCL: $C\frac{dv_o(t)}{dt} + \frac{v_o(t)}{R} + \frac{1}{L}\int v_o(t)dt + i_c(0) = -I_s(t)$

Characteristic equation:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \qquad \frac{1}{LC} = \omega_0^2$$

$$C = \frac{1}{L\omega_0^2} = \frac{1}{4\times10^{-9}\times10^{12}} = 250\,\mu F$$

$$\frac{1}{RC} = 2\zeta\omega_0$$

$$R = \frac{1}{2C\zeta\omega_0} = \frac{1}{250} = 4\,m\Omega$$

(b) $\zeta < 1 \rightarrow$ underdamped

(c) $$Z_{in}(\omega) = \frac{1}{j\omega C + \frac{1}{R} + \frac{1}{j\omega L}} = \frac{j\omega L}{1-\omega^2 LC + j\omega\frac{L}{R}}$$

$$Z_{in}(\omega_0) = \frac{j\,4\times10^{-3}}{1-1+j} = 4\,m\Omega$$

(d) $$V_o = Z_{in}(\omega_0)A\cos\omega_0 t + Z_{in}(2\omega_0)A\cos(2\omega_0 t) + Z_{in}\left(\frac{\omega_0}{2}\right)A\cos(\omega_0 t/2)$$

$$= 4\times10^{-3}A\cos\omega_0 t + 2.219\times10^{-3}A\cos(2\omega_0 t - 56.3^\circ)$$
$$+ 2.219\times10^{-3}A\cos\left(\frac{\omega_0}{2}t + 56.3^\circ\right)$$

Prob 2

KCL: $$\frac{V_{DD} - v_c(t)}{R_L} = C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{R_{NMOS}}$$

$$C\dot{v}_c + \left(\frac{1}{R_N} + \frac{1}{R_L}\right)v_c = \frac{V_{DD}}{R_L}$$

$$10\times10^{-6}\dot{v}_c + 1.1\times10^{-3}v_c = 5\times10^{-4}$$

$$v_p = \frac{5\times10^{-4}}{1.1\times10^{-3}} = 0.4545$$

$$v_{cc} = Ke^{-t/9.1\times10^{-3}}$$

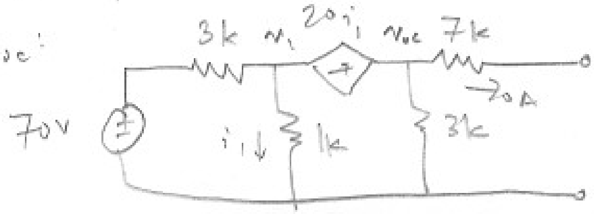
$$v_c(0) = V_{DD} = 5V = 0.4545 + Ke^{-t/9.1\times10^{-3}}$$

$$K = 5 - 0.4545 = 4.545$$

$$\boxed{v_c(t) = 0.4545 + 4.545e^{-t/9.1\times10^{-3}}}$$

EE40 Hw3 p8

v_{oc}:

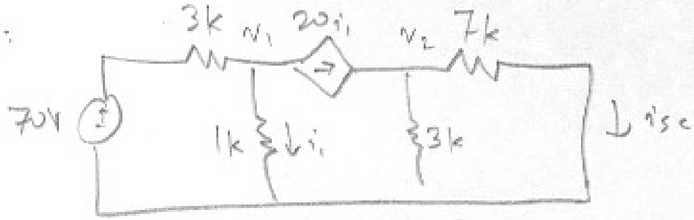


KCL @ v_1 : $\frac{v_1 - 70}{3k} + \frac{v_1}{1k} + \frac{20 v_1}{1k} = 0 \rightarrow 21.33 \times 10^{-3} v_1 + 23.33 \times 10^{-3} = 0$

$v_1 = 1.094\ V$

KCL @ v_{oc} : $\frac{20 v_1}{1k} = \frac{v_{oc}}{3k} \rightarrow v_{oc} = 60 v_1 = \boxed{65.63 V}$

i_{sc}:



$v_1 = 1.094V$ as before

KCL @ v_2 : $\frac{20 v_1}{1k} = \frac{v_2}{7k \| 3k} \rightarrow v_2 = 42 v_1 = 45.94V$

$i_{sc} = \frac{v_2}{7k} = \boxed{6.563 mA}$

$R_{eq} = \frac{v_{oc}}{i_{sc}} = 10 k\Omega$