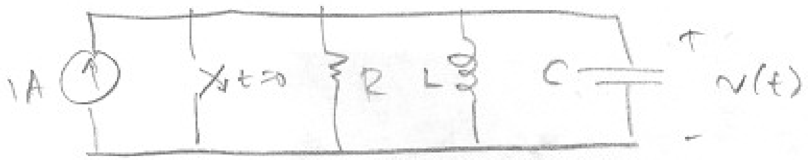
EE40 HW 4 solution p1.

P4.49



KCL: $\frac{v(t)}{R} + \frac{1}{L}\int v(t)\,dt + i_L(0) + C\frac{dv(t)}{dt} = 1A$

$$\frac{d^2v(t)}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{LC}v(t) = 0$$

(a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-5}\,10^{-9}}} = 10^7 = 10\,MHz$

$\alpha = \frac{1}{2RC} = \frac{1}{2\cdot 50\cdot 10^{-9}} = 10^7$

$\zeta = \frac{\alpha}{\omega_0} = \frac{10^7}{10^7} = 1$

(b) Evaluating our KCL eqn @ $t=0^+$:

$\cancel{\frac{0}{R}}^{\,0} + \frac{1}{L}\cancel{\int_0^0 v(t)\,dt}^{\,0} + 0 + Cv'(0^+) = 1$

$v'(0^+) = \frac{1}{C} = 10^9\ V/s$

(c) At steady state ($t\to\infty$), the inductor acts as a short so the particular solution is $v(t)=0$

(d) Critically damped $\rightarrow$ $v(t) = K_1 e^{\omega_0 t} + K_2 t e^{\omega_0 t}$

I.C.: $v(0)=0 \rightarrow K_1 + 0 = 0 \rightarrow K_1 = 0$

$v'(0^+) = 10^9 \rightarrow \cancel{K_1\omega_0}^{\,0} + K_2 + 0 = 10^9 \rightarrow K_2 = 10^9$

$v(t) = 10^9 t e^{-10^7 t}$

P 4.35

$$\frac{v(t)}{R} + \frac{1}{L}\int_0^t v(t)dt + i_L(0) = 5\cos 10t$$

$$\frac{L}{R}\frac{d}{dt}v(t) + v(t) = -50\sin 10t$$

try $v_p(t) = A\cos 10t + B\sin 10t$

$$\frac{d}{dt}v_p(t) = -10A\sin 10t + 10B\cos 10t$$

$$\frac{L}{R}(-A\sin 10t + B\cos 10t) + A\cos 10t + B\sin 10t = -50\sin 10t$$

$$\left(A + \frac{10BL}{R}\right)\cos 10t + \left(B - \frac{10AL}{R}\right)\sin 10t = -50\sin 10t$$

$$A + \frac{10BL}{R} = 0 \qquad B - \frac{10AL}{R} = -50$$

$\rightarrow A = 25 \quad B = -25 \rightarrow v_p(t) = 25\cos 10t - 25\sin 10t$

$\tau = \frac{L}{R} = 0.1s$

$v_c(t) = Ke^{-t/\tau}$

$v(t) = 25\cos 10t - 25\sin 10t + Ke^{-t/\tau}$

@ $t = 0^+$ $\rightarrow$ inductor acts as open circuit

$\therefore$ $v(0^+) = i(0^+)R = 5A \cdot 10\Omega = 50V$

$v(0^+) = 25 - 0 + K = 50 \longrightarrow K = 25$

$\therefore$ $v(t)$ $25\cos 10t - 25\sin 10t + 25e^{-t/\tau}$ for $t > 0$

P6.59

The transfer function is $\dfrac{R}{R + \frac{1}{j\omega C}} = \dfrac{j\omega RC}{1 + j\omega RC}$

$$V_{out} = H(200\pi)\,5\cos(200\pi t) + H(2000\pi)\,5\cos(2000\pi t)$$

$$= 0.4975\cos(200\pi t + 84.29°) + 3.536\cos(2000\pi t + 45°)$$

P6.81

(a) $H(f) = \dfrac{j\omega L + \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C} + R} = \dfrac{1-\omega^2 LC}{1-\omega^2 LC + j\omega RC} = \dfrac{1-4\pi^2 f LC}{1-4\pi^2 f LC + j2\pi f RC}$

(b) MATLAB code would be something like:

```
R=10;
L=0.01;
C=2.533e-8;
logf =3:0.01:5; % log frequencies
f =10.^logf
w= 2*pi*f;
H= (1 - w.*w*L*C)./(1-w.*w*L*c+j*w*R*C);
semilogx(f, 20*log10(abs(H)))
```

(The plot will be a notch filter with the notch at 10^4 Hz and an attenuation of ≈65dB.

(c) At low frequencies, the capacitor acts like an open circuit so there is no current making $v_{out} \approx v_{in}$ or $H(f) \simeq 1$

(d) At high frequencies, the inductor acts like an open circuit making $v_{out} \approx v_{in}$ or $H(f) \simeq 1$

P14.17

(a) $v_- = v_+ = 0$ b/c of negative feedback $i_R = 2mA$ by KCL @ - $(i_- = 0)$

$v_- - v_o = (2mA)(1k\Omega)$ by Ohm's Law

$\therefore v_o = -2V$

(b) $v_- = v_+ = 5V$ $i_R = 2mA$ by KCL @ - $(i_- = 0)$

$v_- - v_o = (2mA)(3k\Omega)$

$\therefore v_o = 5V - 6V = -1V$

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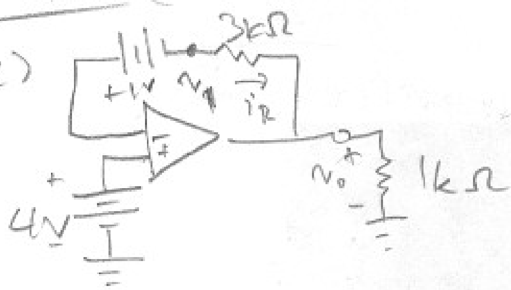
P 14.17 (cont'd)

(c)



$v_- = v_+ = 4V$

$v_1 = v_- - 1V = 3V$

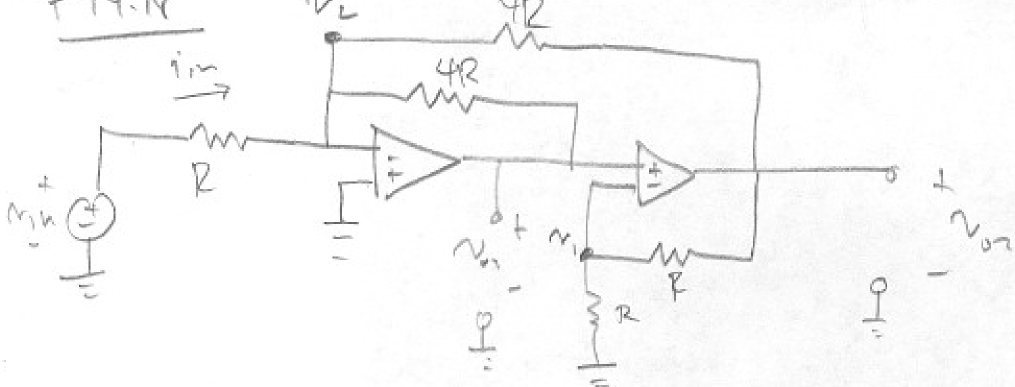
Since $i_- = 0$, $i_R = 0 \rightarrow v_o = v_1 = 3V$

(d) $i_- = 0 \rightarrow i_R = 0$, $v_- = v_+ = 0V$ $\Big\} \rightarrow v_o = v_- = 0V$

(e) $v_- = v_+ = 5V$

$v_o = v_- - 2V = 3V$

P 14.19



$v_{o1} = v_1$ (by neg feedback on OP-AMP 2)

$v_1 = v_{o2}/2$ (by voltage divider)

$\rightarrow v_{o2} = 2v_{o1}$

$v_2 = 0V$ (by neg feedback on OP-AMP 1)

$i_{in} + \frac{v_{o1}}{4R} + \frac{v_{o2}}{4R} = 0$ (KCL @ 2)

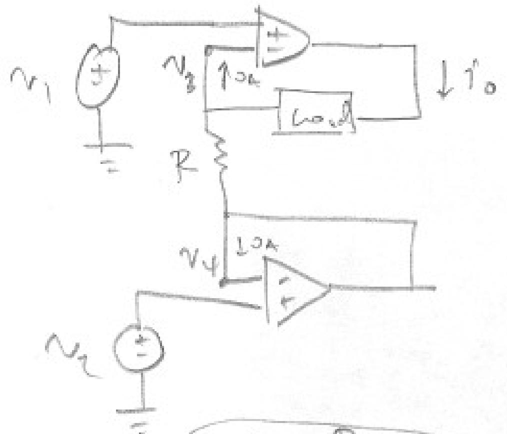
$i_{in} = \frac{v_{in}}{R}$ (ohm's)

$\therefore \frac{v_{in}}{R} = -\frac{3v_{o1}}{4R} \rightarrow \frac{v_{o1}}{v_{in}} = -\frac{4}{3}$ $\quad \frac{v_{o2}}{v_{in}} = 2\left(-\frac{4}{3}\right) = -\frac{8}{3}$

$A_1 = -\frac{4}{3}$ $\quad A_2 = -\frac{8}{3}$

P14.21

(a)

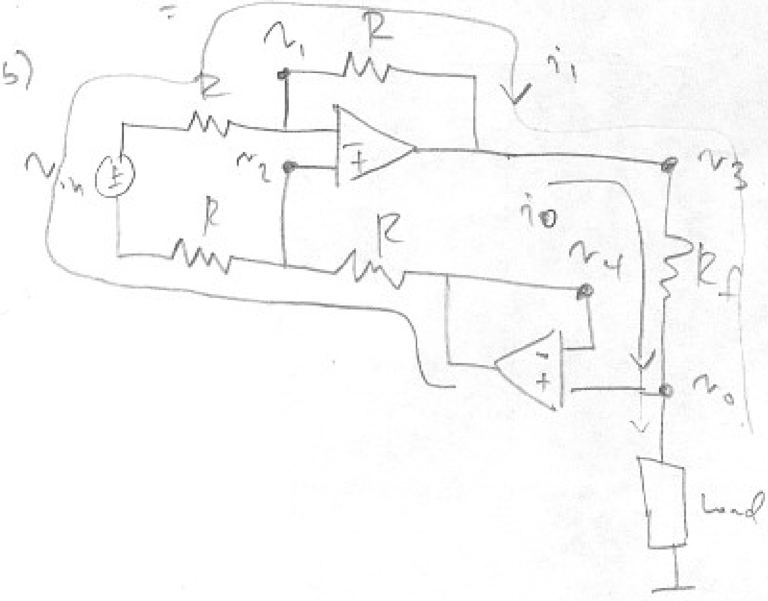


$v_3 = v_1 \quad v_4 = v_2$

$$i_o = \frac{v_3 - v_4}{R} = \boxed{\frac{v_1 - v_2}{R}}$$

Since i_o does not depend on R_L, the output impedance is ∞.

(b)



KVL from v_2 to v_1

$$v_2 - i_1 R + v_{in} - i_1 R = v_1$$

But $v_1 = v_2 \rightarrow i_1 = \frac{v_{in}}{2R}$

KVL from v_4 to v_o

$$v_4 - 4 i_1 R + v_{in} + R_f i_o = v_o$$

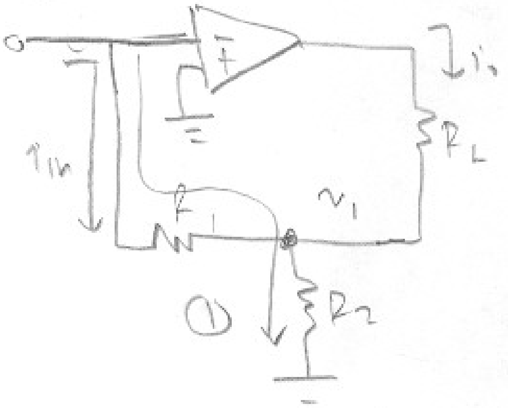
But $v_4 = v_o \rightarrow v_{in} = 4 i_1 R - i_o R_f$

$$= 2 v_{in} - i_o R_f$$

$$\rightarrow \boxed{i_o = -\frac{v_{in}}{R_f}}$$

Again i_o is indep of R_L so output impedance is ∞.

P14.26



KVL on path 1:

$$0V - i_{in} R_1 - (i_{in} + i_o) R_2 = 0V$$

$$\rightarrow i_o = \frac{-i_{in}(R_1 + R_2)}{R_2} = \boxed{-\frac{(R_1 + R_2)}{R_2} i_{in}}$$

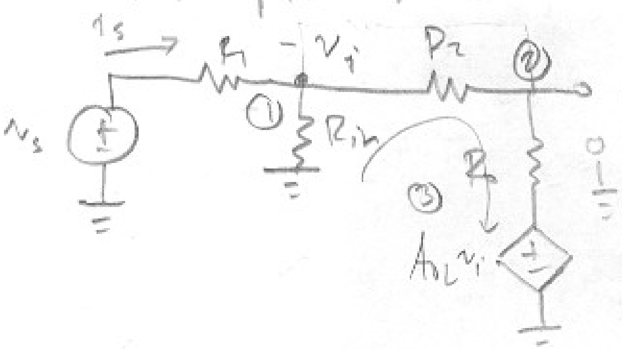
i_o is indep. of R_L so output impedance is infinite

EE40 HW4 soln p.6

P14.40

(a) inverting amp: $\frac{v_o}{v_s} = -\frac{R_2}{R_1} = -10$, ideally

with this model:



KCL @ ① $\frac{v_s + v_i}{R_1} + \frac{v_i}{R_{in}} + \frac{v_o + v_i}{R_2} = 0$

KCL @ ② $\frac{v_o + v_i}{R_2} + \frac{v_o - A_{OL} v_i}{R_o} = 0$

Solving: $\frac{v_o}{v_s} = \frac{-R_2}{R_1\left[1 + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}}\right)\frac{R_o R_2 + R_2^2}{A_{OL} R_2 - R_o}\right]}$

$= -9.9989$

(b) $Z_i = \frac{v_s}{i_s} = R_1 = 1\,k\Omega$ ideally

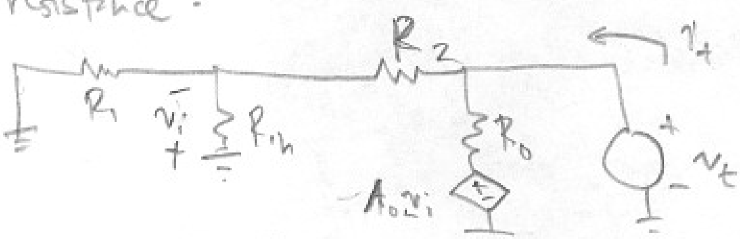
with this model

Ohm's: $v_s = R_1 i_s - v_i$

KVL in ③: $v_i + (R_2 + R_o)\left(\frac{v_i}{R_{in}} + i_s\right) + A_{OL} v_i = 0$

Solving: $Z_{in} = \frac{v_s}{i_s} = R_1 + \frac{R_2 + R_o}{1 + A_{OL} + (R_2 + R_o)/R_{in}} = 1.0001\ k\Omega$

(c) Finding output impedance is just like finding thevenin resistance:



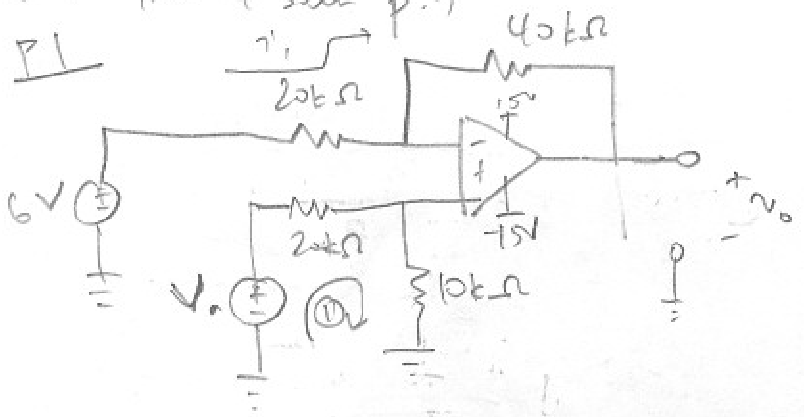
$v_i = -\frac{R_{in} \| R_1}{R_2 + R_{in} \| R_1} v_t$ (voltage divider)

$i_t = \frac{v_t}{R_2 + R_{in} \| R_1} + \frac{v_t - A_{OL} v_i}{R_o}$ (KCL)

$Z_o = \frac{v_t}{i_t} = \frac{1}{\frac{1}{R_2 + R_{in} \| R_1} + \frac{1}{R_o} + \frac{A_{OL}(R_{in} \| R_1)}{R_o(R_2 + R_{in} \| R_1)}} = 2.75\ m\Omega$

EE40 HW 4 soln p.7

P1



$$v_+ = \frac{v_a}{3} \quad \text{(by KVL on ① — voltage divider)}$$

$$v_- = v_+$$

$$i_1 = \frac{6V - v_-}{20k} \quad \text{(ohm's)}$$

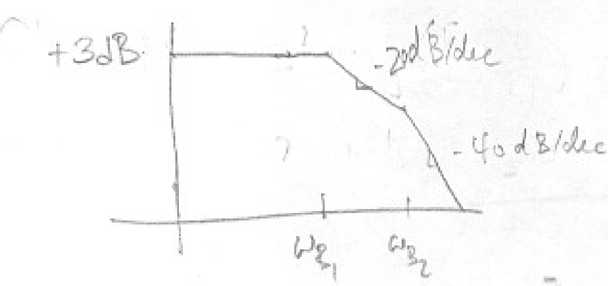
$$v_o - v_- = -i_1(40k) = 12V - \frac{2}{3}v_a \qquad \frac{v_o}{v_a} = -\frac{2}{3} \quad \text{(w/ 12V offset)}$$

P2

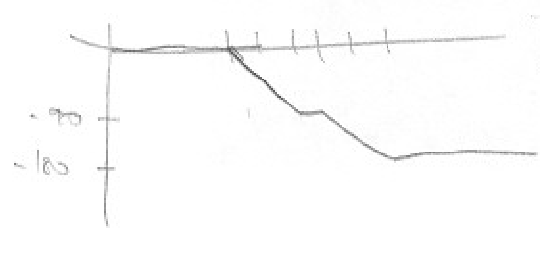
$$\frac{v_{in}}{j\omega L + R} + v_o j\omega C + \frac{v_o}{2R} = 0$$

$$\frac{v_o}{v_{in}} = \frac{-1}{(R + j\omega L)\left(\frac{1}{2R} + j\omega C\right)} = \frac{-\frac{1}{R}2R}{(1 + j\omega L/R)(1 + j\omega 2RC)}$$

$$= \frac{-2}{(1 + j\omega/(\frac{R}{L}))(1 + j\omega/(\frac{1}{2RC}))} \qquad \text{let } \omega_{B_1} = \frac{R}{L} \quad \omega_{B_2} = \frac{1}{2RC}$$



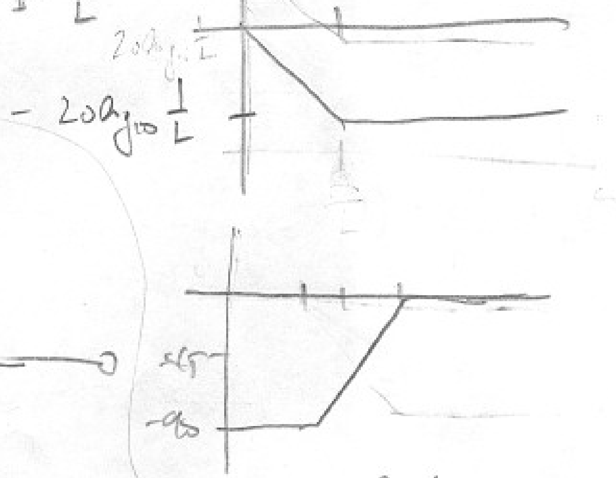
Bode plots vary depending on how close ω_B's are...



P3

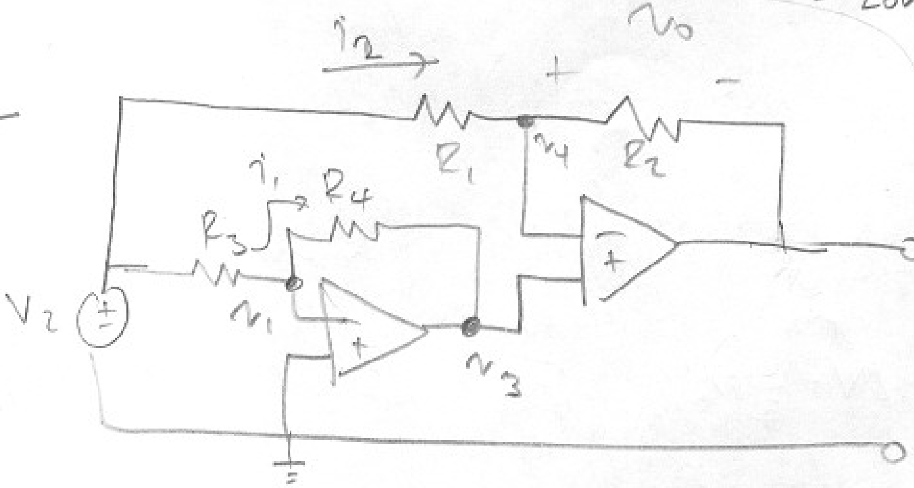
$$\frac{v_{in}}{j\omega} = \frac{v_o}{R+j\omega L} \rightarrow \frac{v_o}{v_{in}} = \frac{R+j\omega L}{j\omega} = \frac{1}{L}\frac{1+j\omega L/R}{j\omega L/R}$$

$$= \frac{1}{L}\left(\frac{j\omega/(\frac{R}{L})}{1+j\omega/(\frac{R}{L})}\right)^{-1} \leftarrow \text{HPF w/ } \omega_B = \frac{R}{L}$$



note $\log\frac{1}{x} = -\log x$
so we just
invert the transfer
function

P4



$$\left.\begin{aligned} v_1 &= 0 \\ i_1 &= \frac{v_2}{R_3} \end{aligned}\right\} \longrightarrow v_3 = -\frac{R_4}{R_3}v_2$$

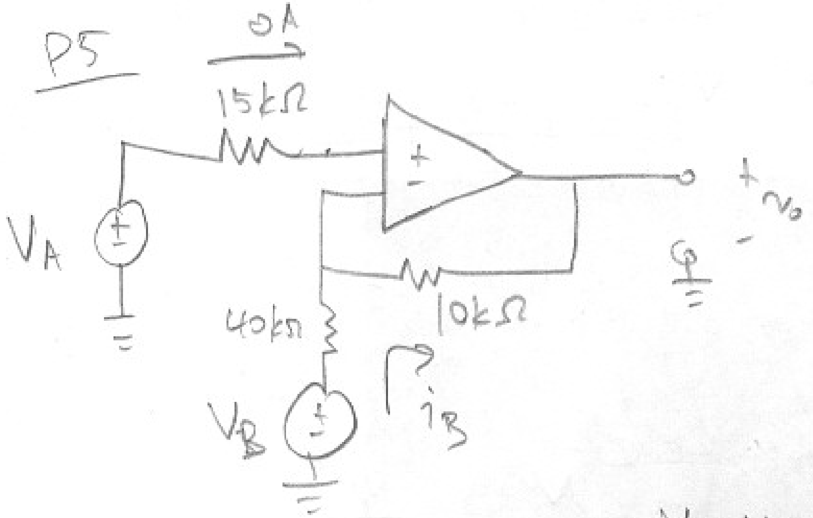
$$v_4 = v_3 = -\frac{R_4}{R_3}v_2$$

$$i_2 = \frac{v_2 - v_4}{R_1} = v_2\frac{\left(1+\frac{R_4}{R_3}\right)}{R_1}$$

$$v_o = i_2 R_2 = \frac{R_2}{R_1}\left(1+\frac{R_4}{R_3}\right)$$

EE40 HW4 soln p9.

P5

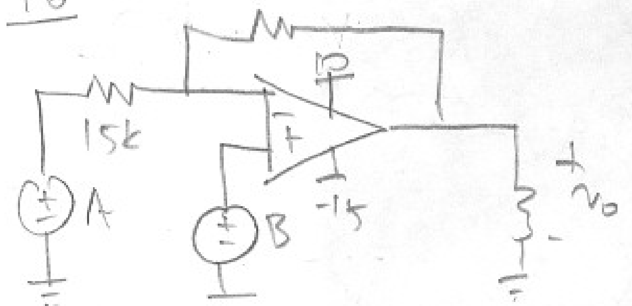


$v_- = v_+ = V_A \rightarrow i_B = \dfrac{V_B - V_A}{40\,k\Omega}$

$v_o = V_A - i_B \, 10k\Omega$

$= \dfrac{5V_A - 3V_B}{4}$

P6



By KCL @ (−)

$\dfrac{A - B}{15k} = \dfrac{B - v_o}{25k}$

@ $V_o = 10V \rightarrow A = \frac{8}{5}B - 6V$

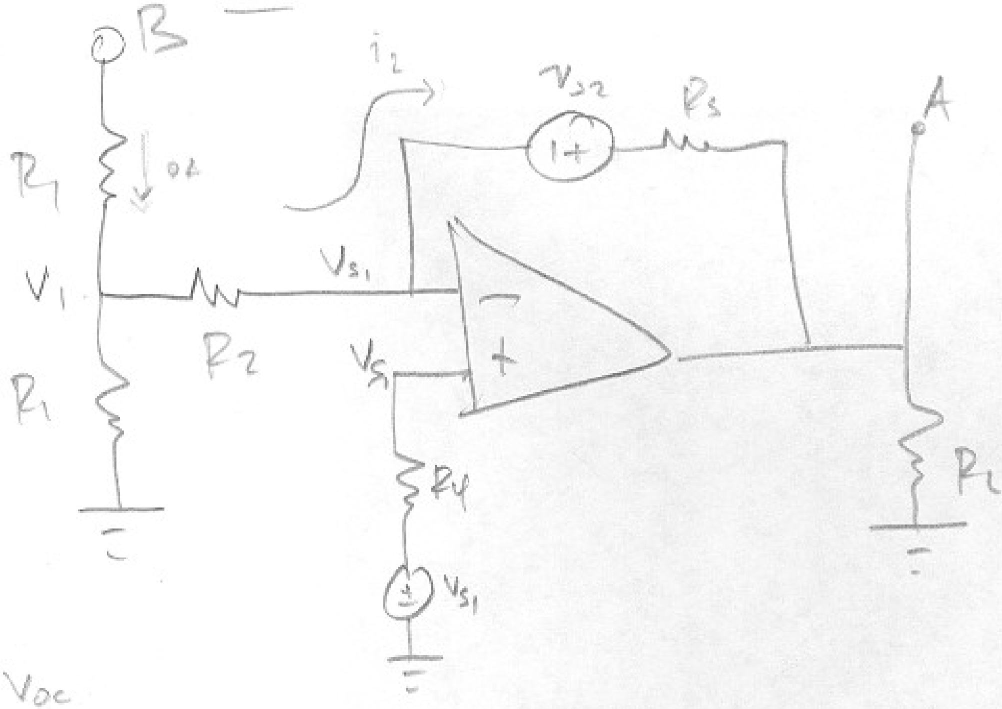
@ $V_o = -15V \rightarrow A = \frac{8}{5}B + 9V$

if $A = 2B \rightarrow 2B = \frac{8}{5}B - 6V \rightarrow B = -15V$

$2B = \frac{8}{5}B + 9V \rightarrow B = 22.5V$

EE40 HW 4 sln p 10

P7 V_{oc}:



V_{oc}

$$V_{AB} = V_{s1} + V_{s2} - i_2 R_3$$

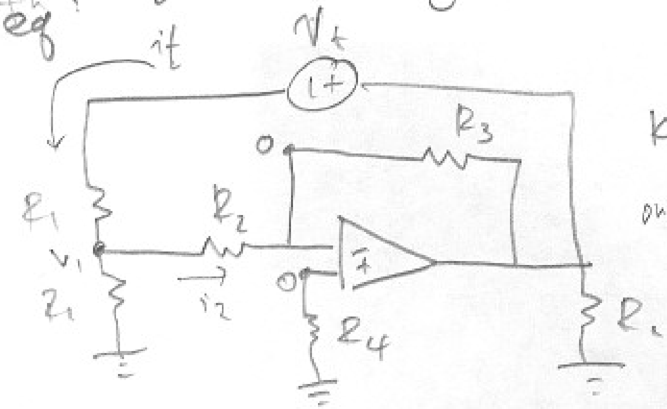
$$i_2 = \frac{-V_{s1}}{R_1 + R_2}$$

$$V_{oc} = V_{s1} + V_{s2} + \frac{V_{s1} R_3}{R_1 + R_2} = \boxed{\left(\frac{R_1 + R_2 + R_3}{R_1 + R_2}\right) V_{s1} + V_{s2} = V_{th}}$$

EE40 Hw4 soln p11

P7 (cont'd)

R_{eq}^{th}: (zero sources apply v_t)



KCL @ v_1

$$i_t = \frac{v_1}{R_1} + \frac{v_1}{R_2} \rightarrow v_1 = \frac{i_t(R_1+R_2)}{R_1 R_2}$$

KVL:

$$v_t - i_t R_1 + i_2(R_2+R_3) = 0$$

ohms:

$$i_2 = \frac{v_1}{R_2}$$

substituting:

$$v_t - i_t R_1 + \frac{i_t(R_1+R_2)(R_2+R_3)}{R_1 R_2^2} = 0$$

$$\rightarrow \boxed{R_{eq} = \frac{v_t}{i_t} = R_1 - \frac{(R_1+R_2)(R_2+R_3)}{R_1 R_2^2}}$$