EE40 HW6 soln

12.15

$$i_D = K(v_{GS} - V_T)^2$$

$$v_{GS} = V_T \pm \sqrt{\frac{i_D}{K}} = -2V \pm 2V = 0V, -4V$$

But 0V would not be saturation, so $\boxed{v_{GS} = -4V}$

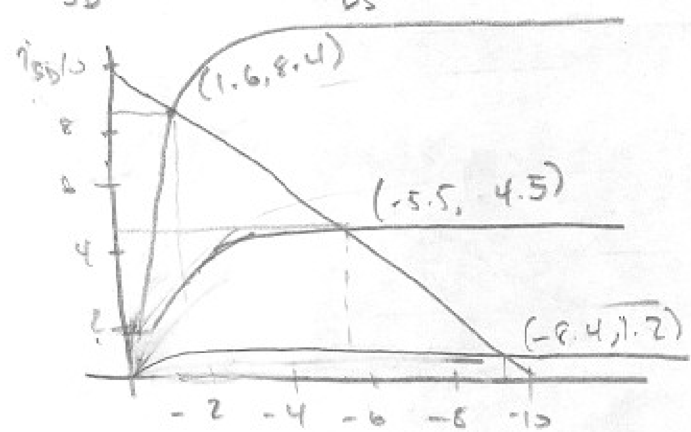
12.24

$v_{GS} = 7V + \sin 2000\pi t - 10V = -3V + \sin 2000\pi t$

Because sin can range between 1V and -1V,

$v_{GS\,MAX} = -2V$, $v_{GS\,min} = -4V$ $v_{GSQ} = -3V$

$i_{SD} = 10A + v_{DS}$



$v_{out} = 1k\, i_{SD}$

$v_{o\,max} = 8.4\ V$

$v_{o\,min} = 1.2\ V$

$v_{o\,Q} = 4.5\ V$

12.30

$$I_{DQ} = \frac{20V - 8V - 2V}{2k\Omega} = 5mA$$

$R_S = 2V / I_{DQ} = \boxed{400\Omega}$

Assuming saturation ($V_{DS} = 8V$)

$I_{DQ} = \frac{1}{2}\, 50\times10^{-6}\, \frac{600}{20} (V_{GS} - 1)^2 \rightarrow V_{GS} = -1.582,\ 3.582$

not saturation

(Check $V_{DS} > V_{GS} - V_T$)

$V_G = V_{GS} + 2V = 5.582V$

$V_G = 20V \frac{1M\Omega}{1M\Omega + R_1} \rightarrow R_1 = \frac{1M\Omega}{V_G/20V} - 1M\Omega = \boxed{2.583\ M\Omega}$

12.37

$$i_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(v_{GS} - V_T)v_{DS} - v_{DS}^2\right]$$

$$g_m = \frac{\partial i_D}{\partial v_{GS}}\Big|_Q = \mu_n C_{ox}\frac{W}{L} v_{DSQ}$$

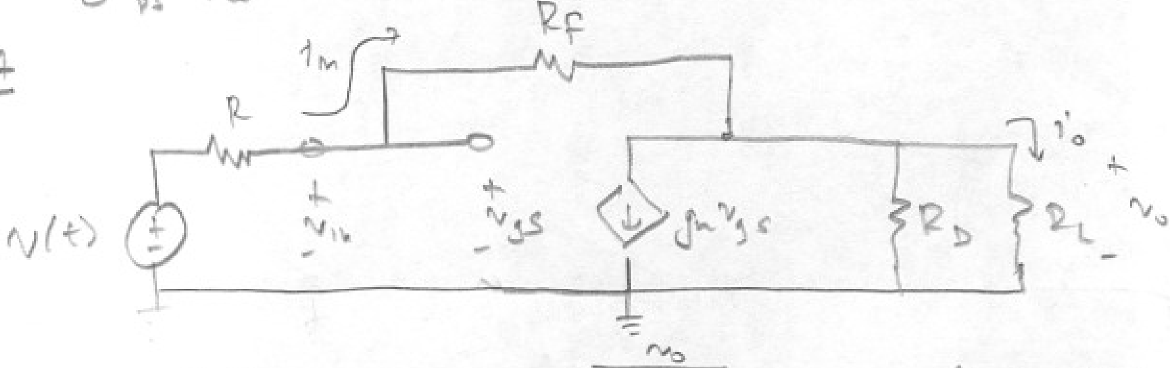
12.40

$$g_m = \frac{\partial i_D}{\partial v_{GS}}\Big|_Q = 3e^{v_{GSQ}} = 3e = 8.155\,mS$$

$$\frac{1}{r_d} = \frac{\partial i_D}{\partial v_{DS}}\Big|_Q = 0.02\,v_{DSQ} = 0.2\,mS \rightarrow r_d = \frac{1}{0.2\,mS} = 5\,k\Omega$$

12.47

(a)



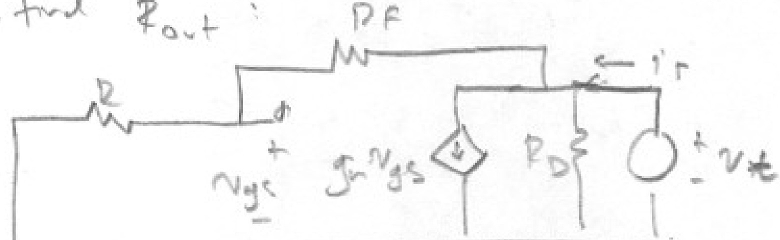
(b) KCL : $i_{in} = g_m v_{gs} + \frac{v_o}{(R_D || R_L)} \rightarrow v_o = (R_D || R_L)(i_{in} - g_m v_{gs})$

$$i_{in} = \frac{v_{in} - v_{out}}{R_f} \qquad \rightarrow v_o = (R_D || R_L)\left(\frac{v_{in} - v_o}{R_f} - g_m v_{gs}\right)$$

Solving: $A_v = \frac{v_o}{v_{in}} = \frac{(R_D || R_L) - g_m (R_D || R_L) R_f}{(R_D || R_L) + R_f}$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{R_f}{1 - A_v}$$

To find R_{out} :



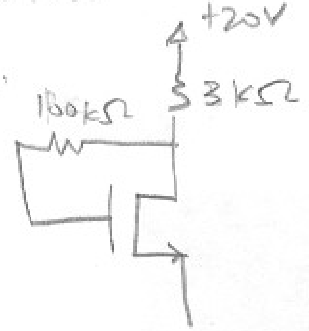
$$v_g = v_t \frac{R}{R + R_f} \quad \text{(voltage divider)}$$

$$R_{out} = \frac{v_t}{i_t} = \frac{1}{\frac{1}{R_D || (R + R_f)} + \frac{g_m R}{R + R_f}}$$

KCL : $i_t = \frac{v_t}{R_D || (R + R_f)} + g_m v_{gs} = v_t\left(\frac{1}{R_D || (R + R_f)} + \frac{g_m R}{R + R_f}\right)$

2.47 (cont'd)

(c) DC:



Since there is no current into the gate: $V_{GSQ} = V_{DSQ}$

Thus $V_{DSQ} > V_{GSQ} - V_T \rightarrow$ saturation

$$I_{DQ} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DS} - V_T)^2$$

Load line: $I_{DQ} = (20V - V_{DS})/3k\Omega$

Solving: $V_{DSQ} = 7.08,\ 2.59$ ← $< V_T$

$$I_{DQ} = K(V_{DSQ} - V_T)^2 = 4.31mA$$

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q = \frac{\partial}{\partial v_{GS}} K(\underbrace{V_{GS}}_{V_{GS}=V_{DS}} - V_T)^2\Big|_Q = 2K(V_{GSQ} - V_T) = 4.16\ mS$$

(d) $A_V = -9.37$

$R_{in} = 9.64k\Omega$

$R_o = 414\Omega$

(e) $v_o(t) = A_v v_{in}(t) = A_v \frac{R_{in}}{R + R_{in}} v(t) = -0.164\sin(2000\pi t)$

(f) inverting $(A_v < 0)$

2.51

Assuming saturation:

$$I_{DQ} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GSQ} - V_T)^2$$

$$\rightarrow V_{GSQ} = V_T + \sqrt{\frac{2 I_{DQ} L}{\mu_n C_{ox} W}} = 3.236V$$

$$V_G = V_{DD}\frac{R_2}{R_1 + R_2} = 10V$$

$$V_G = V_{GSQ} + R_S I_{DQ} \rightarrow R_S = \frac{V_G - V_{GSQ}}{I_{DQ}} = \boxed{3.382k\Omega}$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_{DQ}} = 1.789\ mS$$

$$A_v = \frac{v_o}{v_{in}} = \frac{g_m(R_L \| R_S \| r_d)}{1 + g_m(R_L \| R_S \| r_d)} = \boxed{0.6922}$$

12.51 cont'd

$R_{in} = R_1 \| R_2 = 666.7\,k\Omega$

$R_o = \dfrac{1}{g_m + \frac{1}{R_S} + \frac{1}{r_d}} = 386.9\,\Omega$

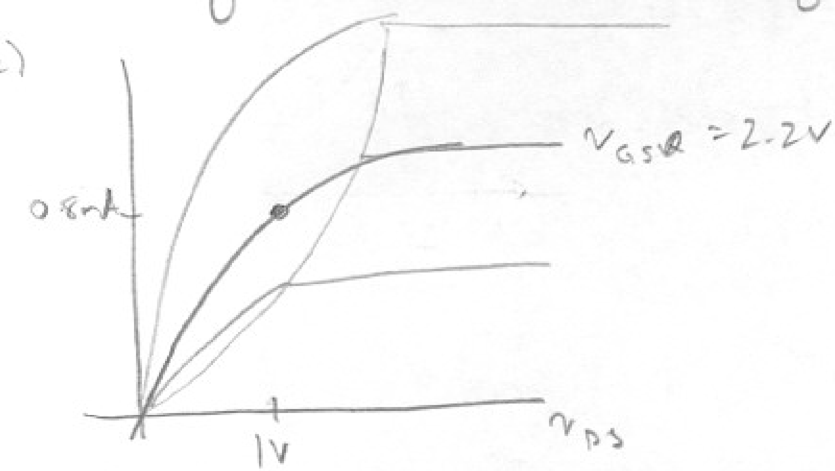
Pb1

(a) $V_G = V_{DD} \dfrac{R_2}{R_1 + R_2} = 3V$

(b) $V_{GS} = V_G - I_{DS} R_S$

Plotting the load line, we get $V_{GS} = 2.2V$ $I_{DS} = 0.8mA$

(c)



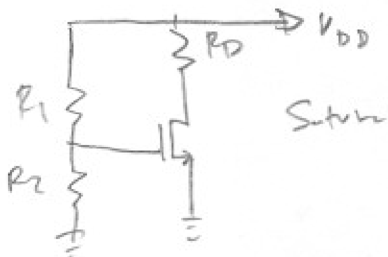
(d) $V_{DSQ} = V_{DD} - I_{DSQ}(R_D + R_S) = 1V$

(e) No

(f) $v_o = V_{DD} - I_{DS} R_D = 1.8V$

Pb2

(a)



$V_{GSQ} = V_{DD} \dfrac{R_2}{R_1 + R_2} = 3V$

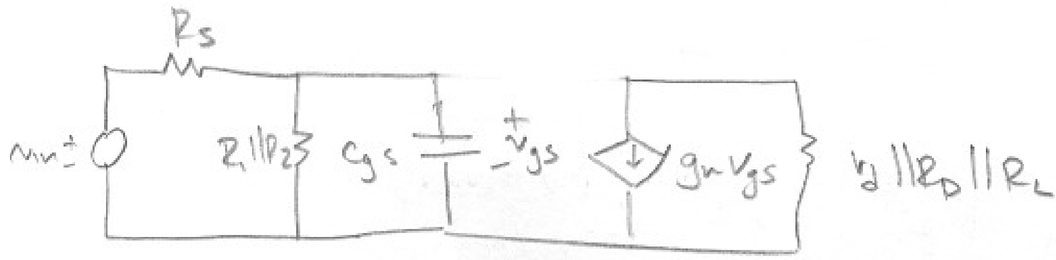
Saturation:

$I_{DSQ} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GSQ} - V_T)^2$

$= 4mV$

$V_{DSQ} = V_{DD} - I_{DSQ} R_D = 7V$

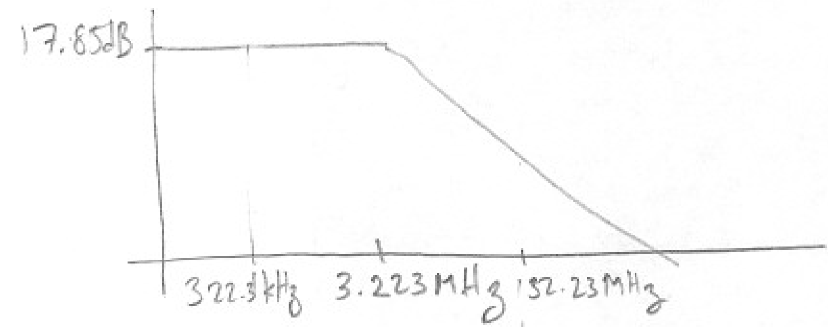
Pb 2 (cont'd)
(b)

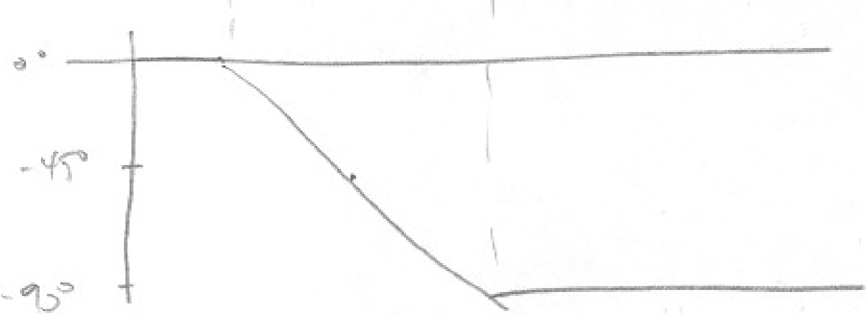


$$\frac{v_g}{v_{in}} = \frac{R_1 \| R_2 \| \frac{1}{j\omega C_{gs}}}{R_s + R_1 \| R_2 \| \frac{1}{j\omega C_{gs}}} = \frac{R_G}{R_G + R_s + j\omega R_G R_s C_{gs}} = \frac{8\times10^5}{8.1\times10^5 + j\omega/25000}$$

(c) $$\frac{v_o}{v_{in}} = -g_m v_{gs}(r_d \| R_D \| R_L) = \frac{5.816\times10^6}{8.1\times10^5 + j\omega/25000}$$

$$= 7.183 \frac{1}{1 + j\omega/2\pi\cdot 3.223\,\mathrm{MHz}}$$





Pb 3

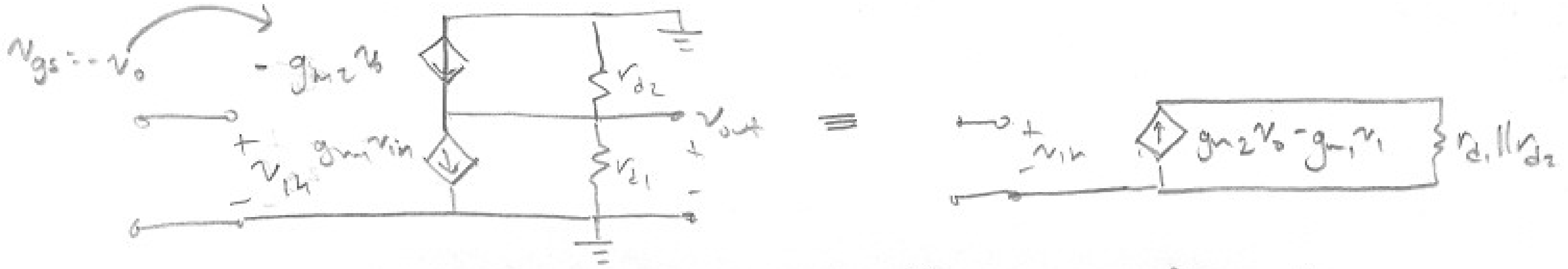
(a)

$$g_m = \frac{\partial i_{SD}}{\partial v_{GS}} = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_T)$$

$$g_m^2 = 2\mu_n C_{ox} \frac{W}{L} I_D$$

$$\rightarrow g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_{SD}}$$

$$\frac{1}{r_d} = \frac{\partial i_{SD}}{\partial v_{DS}} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_T)^2 \lambda = \lambda I_{SD} \rightarrow r_d = \frac{1}{\lambda I_{SD}}$$



(c)

$$\frac{v_o}{r_{d1}\|r_{d2}} = (-g_{m2}v_o - g_{m1}v_{in})$$

$$\rightarrow \boxed{\frac{v_o}{v_{in}} = -g_{m1}\left(r_{d1}\|r_{d2}\|\frac{1}{g_{m2}}\right)}$$

Pb 4

$$\frac{V_1}{V_{in}} = \frac{j\omega RC_1}{1+j\omega R_1C_1} \quad \text{(HPF)}$$

$$V_{OUT} = \frac{-G_m V_1}{\frac{1}{R_2}+j\omega C_2} = -\frac{G_m V_1 R_2}{1+j\omega R_2 C_2} \rightarrow \frac{V_{OUT}}{V_1} = -G_m R_2 \frac{1}{1+j\omega R_2 C_2}$$

$$\frac{V_{out}}{V_{in}} = -G_m R_2 \frac{j\omega R_1 C_1}{1+j\omega R_1 C_1} \frac{1}{1+j\omega R_2 C_2} = -10 \frac{j\omega/1000}{(1+j\omega/1000)(1+j\omega/1\times10^6)}$$