Lecture 2, slide 2:

$v_{ab} = -1V$

$v_{ca} = -2V$

$v_{cb} = v_{ca} + v_{ab} = -3V$

Slide 26:

Σ current leaving $-$ Σ currents entering $= 0$

$10mA + i + 15mA - 5mA = 0$

$i = -20mA$

Slide 28:

Using generalized KCL, $-i + 5\mu A + 2\mu A = 0$

$i = 7\mu A$

Slide 33:

Path 1: $-V_a + V_2 + V_b = 0$

Path 2: $-V_b - V_3 + V_c = 0$

Path 3: $-V_a + V_2 - V_3 + V_c = 0$

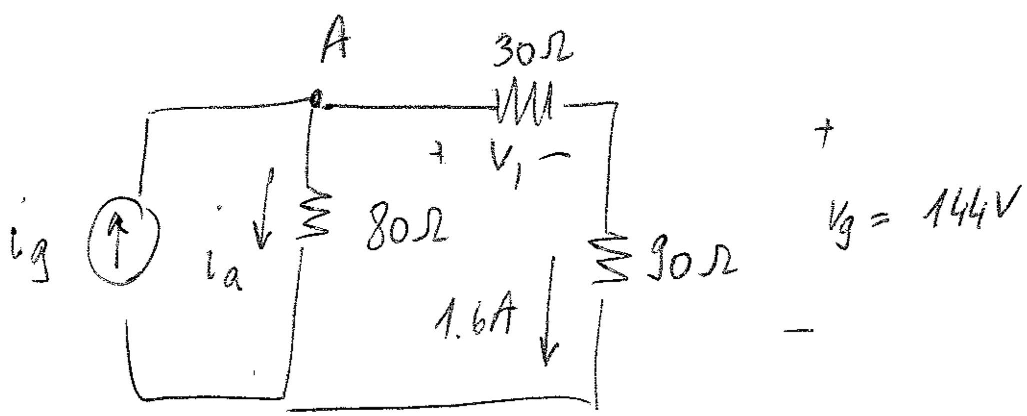
Slide 39:

$$V_2 = \frac{R_2}{R_1 + R_2 + [(R_3 + R_4) \parallel R_5]} \cdot V_{SS}$$

$$\neq \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

Slide 43:

$$v_g = 90\Omega \cdot 1.6A = 144V$$

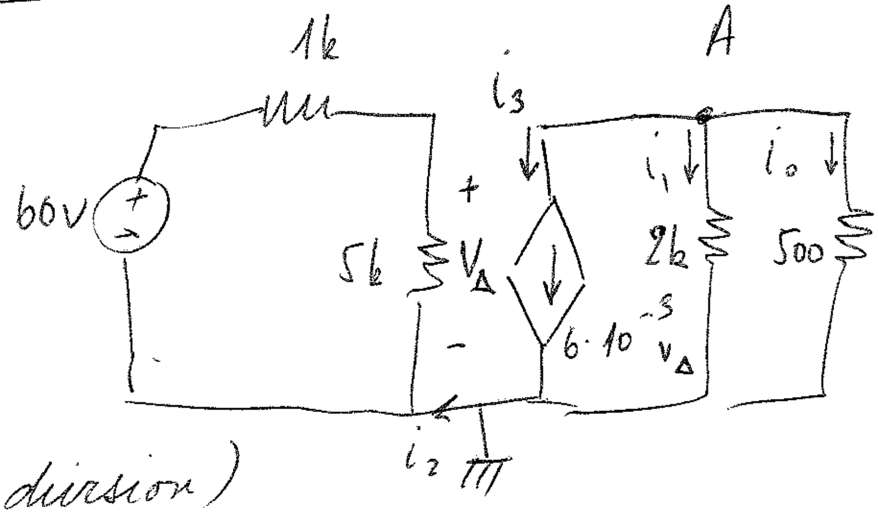


$$V_1 = 1.6A \cdot 30\Omega = 48V$$

$$\Rightarrow V_A = V_1 + v_g = 48V + 144V = 212V$$

$$i_a = \frac{V_A}{R_{80\Omega}} = \frac{212}{80} A$$

Lecture 3 — Slide 3:



$$i_2 = 0$$

$$V_\Delta = \frac{5k}{5k + 1k} \cdot 60V$$

$$= 50V \quad \text{(Voltage division)}$$

$$i_3 = 50V \cdot 6 \cdot 10^{-3} = 0.3A$$

$$2k\Omega \,//\, 500\Omega = 400\Omega$$

$$\Rightarrow V_A = -i_3 \cdot 400\Omega = 12V$$

$$i_1 = \frac{12V}{2k} = 0.06A$$

$$i_0 = \frac{12V}{500} = 0.24A$$

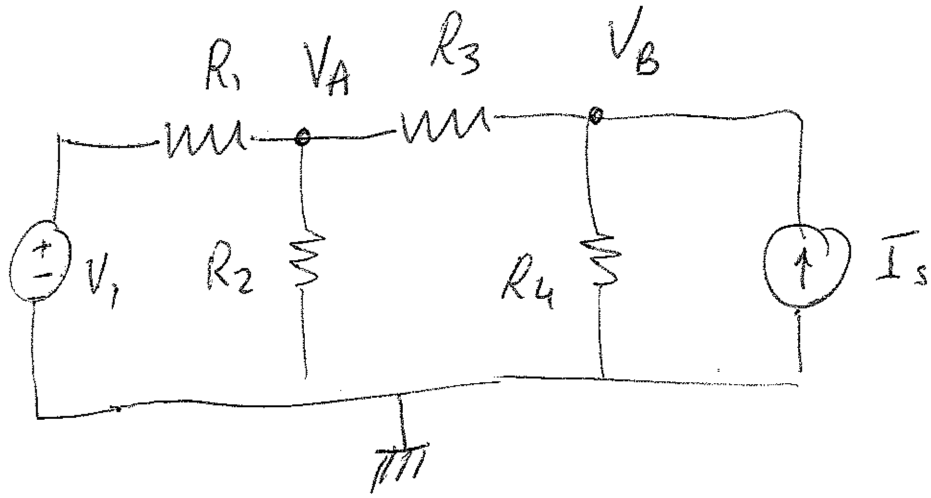
Slide 5:

$$R_{eq} = \left[(R_1 + R_2) \parallel R_3\right] + \left[(R_4 + R_5) \parallel R_6\right]$$

$$= 8k\Omega$$

$$I = \frac{7V}{8k\Omega}$$

Slide 7:



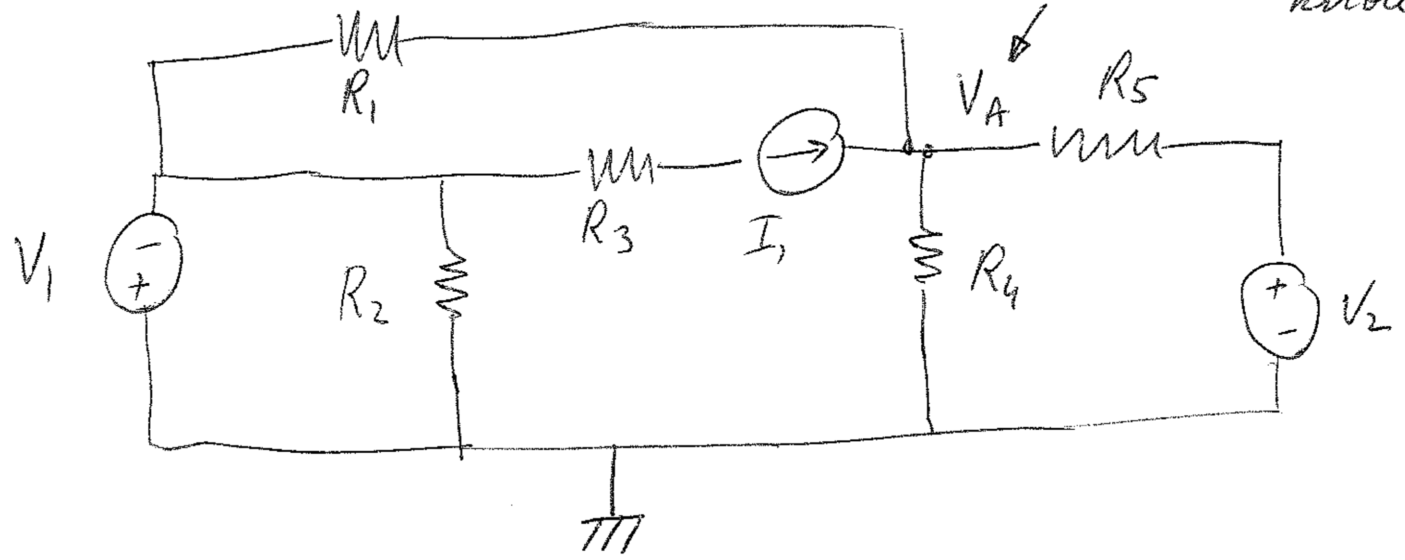
KCL @ V_A: $\quad \dfrac{V_A - V_1}{R_1} + \dfrac{V_A}{R_2} + \dfrac{V_A - V_B}{R_3} = 0$

KCL @ V_B: $\quad \dfrac{V_B - V_A}{R_3} + \dfrac{V_B}{R_4} + (-I_s) = 0$

$\rightarrow$ 2 eq. 2 unknown.

Slide 8:

only V we don't know

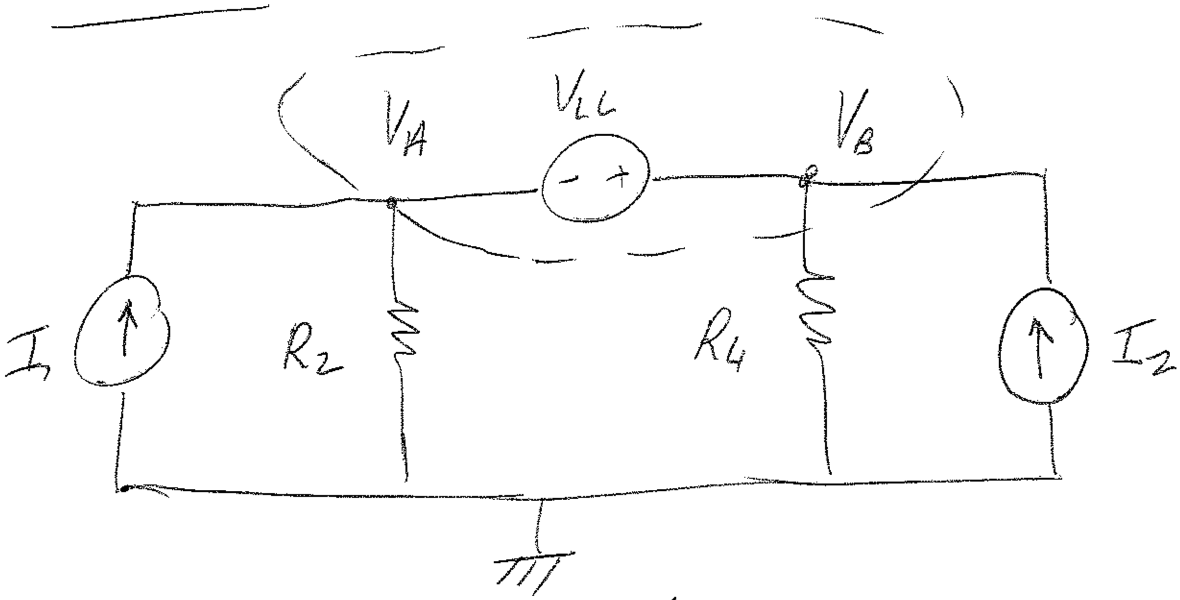


KCL @ V_A:

$$\frac{V_A + V_1}{R_1} + \frac{V_A}{R_4} + \frac{V_A - V_2}{R_5} + (-I_1) = 0A$$

one equation, one unknown.

Slide 11:

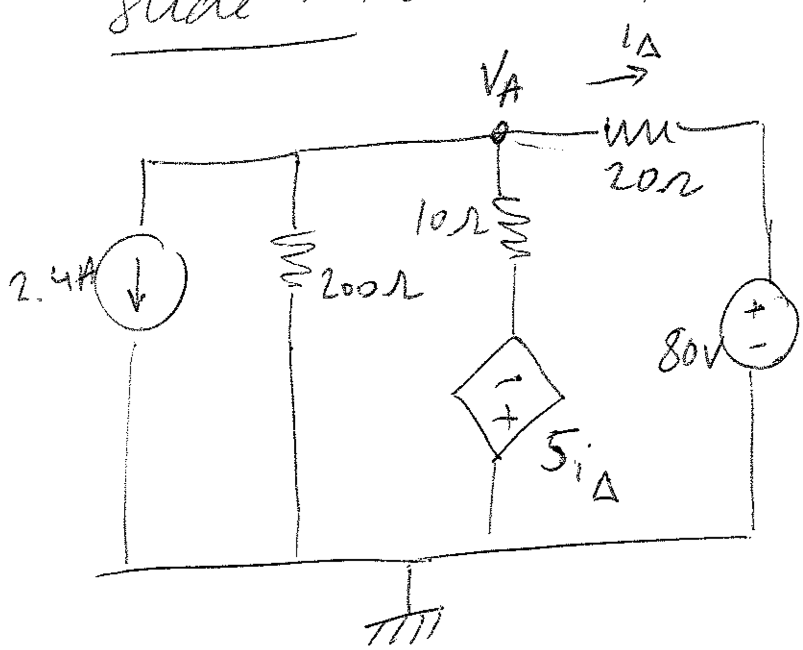


KCL @ supernode:

$$\begin{cases} -I_1 + \dfrac{V_A}{R_2} + \dfrac{V_B}{R_4} - I_2 = 0 \\ V_B = V_A + V_{LL} \end{cases}$$

2 eq., 2 unknown

Slide 14:

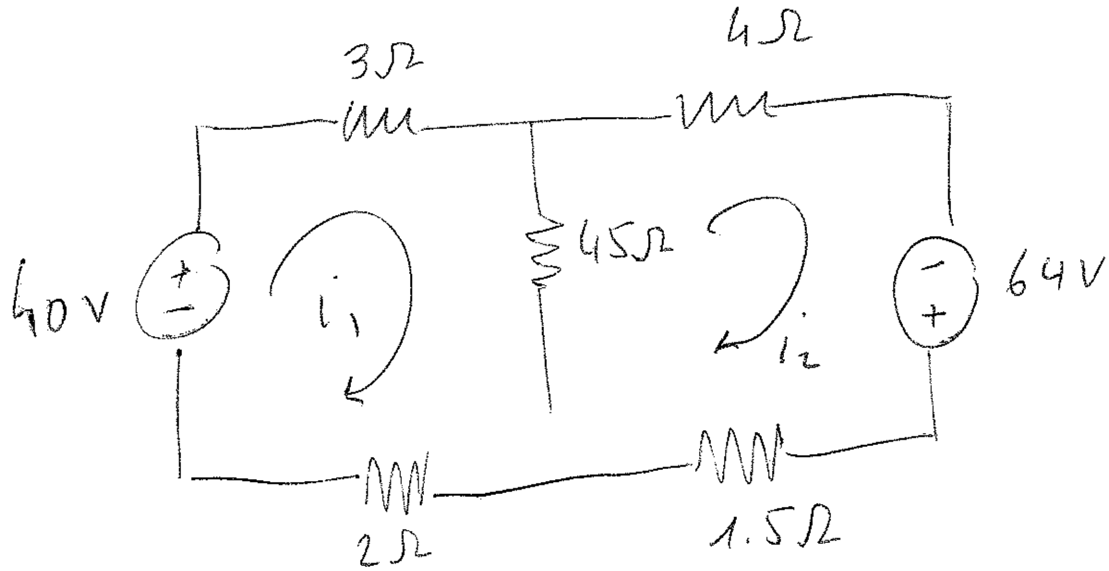


KCL @ V_A

$$\begin{cases} 2.4A + \dfrac{V_A}{200} + \dfrac{V_A - 80}{20} + \dfrac{V_A - (-5i_\Delta)}{10} = 0 \\ i_\Delta = \dfrac{V_A - 80}{20} \end{cases}$$

2 equations
2 unknown

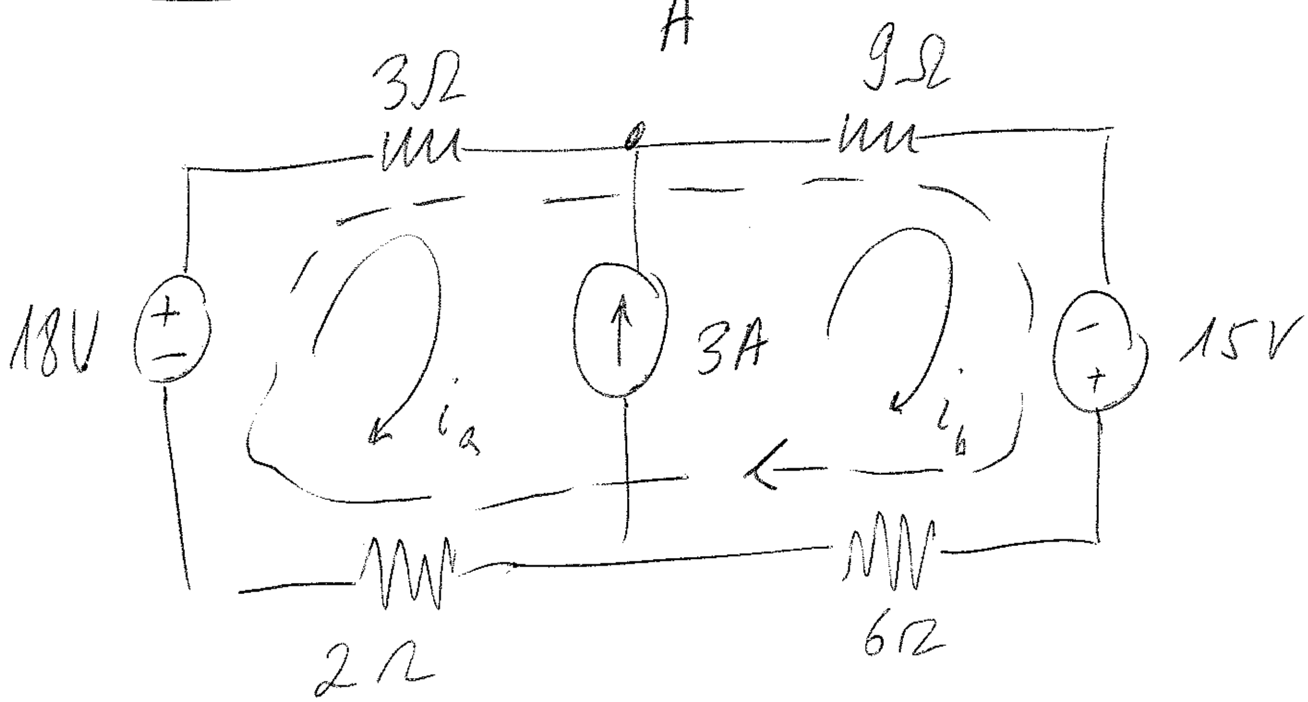
Slide 16



$$\begin{array}{l} \text{2eq} \\ \text{2uk} \end{array} \left\{ \begin{array}{l} \text{KVL @ } i_1 : \\ -40V + i_1 \cdot 3\Omega + (i_1 - i_2)\, 45\Omega + i_1 \cdot 2 = 0V \\ \text{KVL @ } i_2 \\ -64V + i_2 \cdot 1.5\Omega + (i_2 - i_1)\, 45\Omega + i_2 \cdot 4\Omega = 0 \end{array} \right.$$

uk = unknown

Slide 18:



KVL @ supermesh:

$$-18V + i_a \cdot 3\Omega + i_b \cdot 9\Omega - 15V + i_b \cdot 6\Omega + i_a \cdot 2\Omega = 0V$$

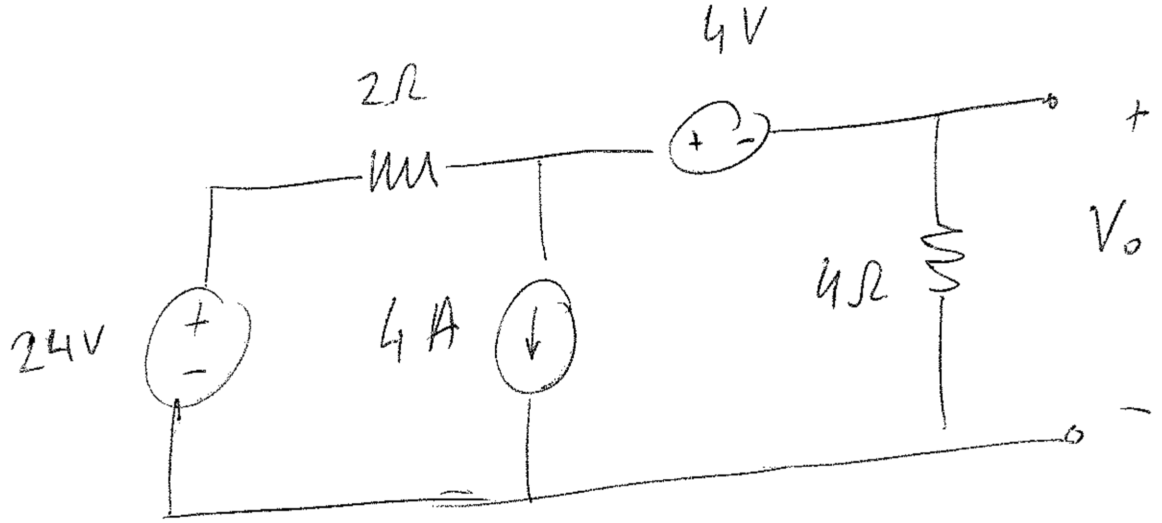
also, KCL @ A:

$$-i_a - 3A + i_b = 0$$

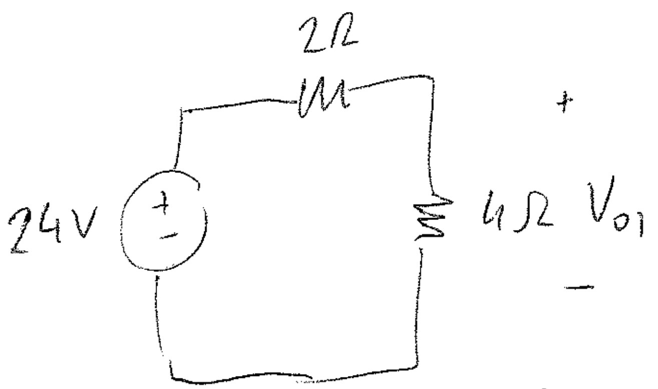
2 eq / 2 uk

uk = unknown

Slide 23:



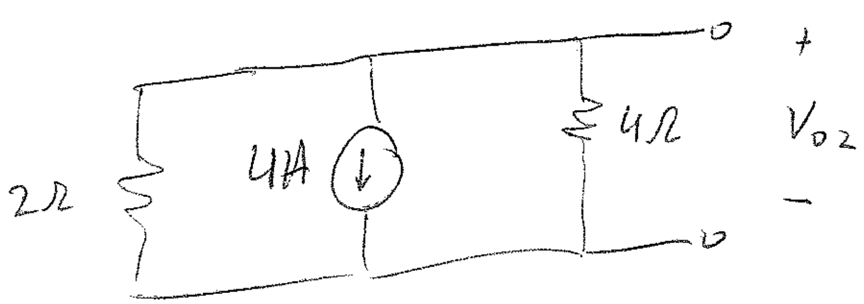
contribution from 24V source. Turn all other sources off:



$$V_{o_1} = \frac{4}{2+4} \cdot 24V$$

$$= 16V$$

contribution from 4A source:



$$R_{eq} = \left(\frac{1}{2} + \frac{1}{4}\right)^{-1}$$

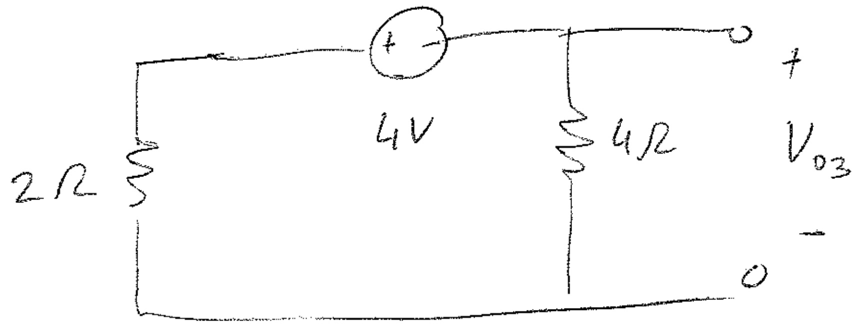
$$= \frac{4}{3}\,\Omega$$

$$\Rightarrow V_{o2} = -4A \cdot R_{eq}$$

$$= -\frac{16}{3}\,V$$

(... cont'd)

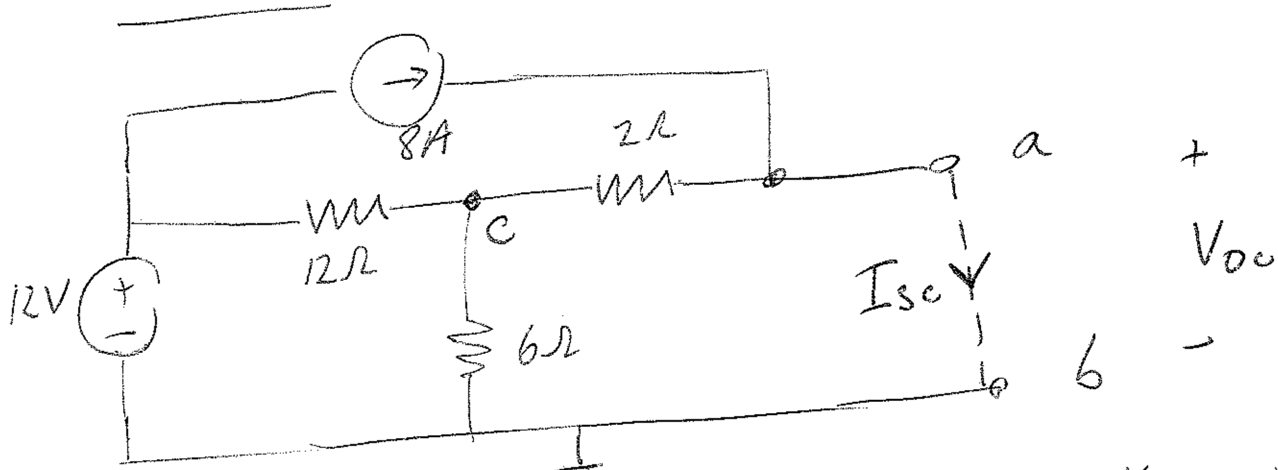
Contribution from 4V source:



$$V_{o3} = -4V \cdot \frac{4\Omega}{6\Omega} = -\frac{8}{3} V$$

$$V_o = \sum V_{o_i} = 16V - \frac{16}{3} V - \frac{8}{3} V = 8V$$

Slide 30:



$V_{Th} = V_{oc}$: KCL @ c: $\frac{V_c - 12}{12} + \frac{V_c}{6} + \frac{V_c - V_a}{2} = 0$

KCL @ a: $-8A + \frac{V_a - V_c}{2} = 0$

→ solve for V_a

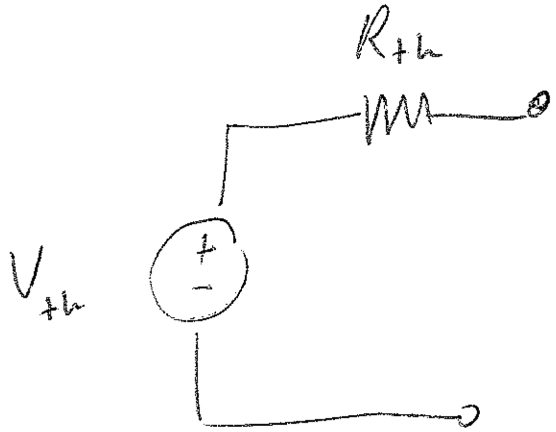
$V_{th} = V_a$

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

I_{sc}: KCL @ c:

$$\frac{V_c - 12}{12} + \frac{V_c}{6} + \frac{V_c}{2} = 0$$ solve for V_c

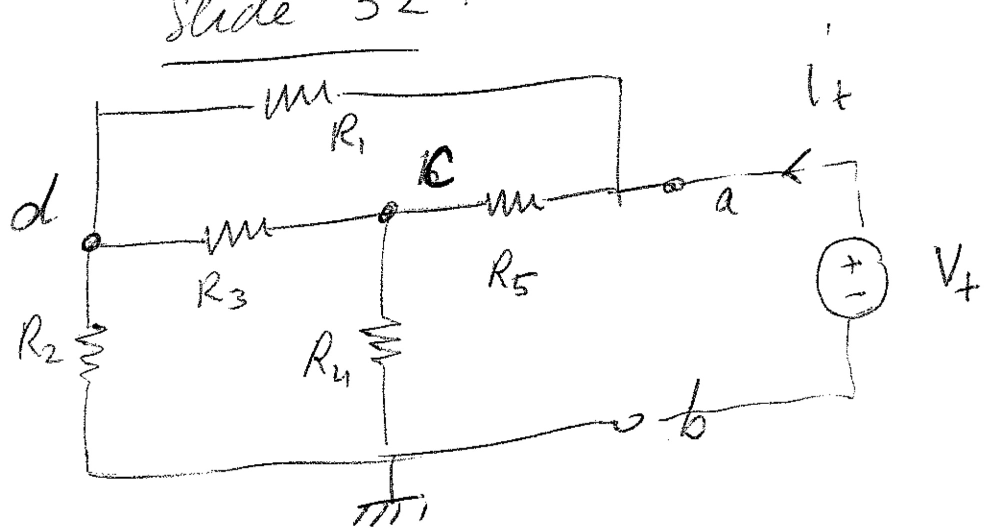
$$I_{sc} = 8A + \frac{V_c}{2\Omega}$$ [KCL @ a]



$$V_{th} = V_{oc}$$

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

slide 32:



solve for V_t & i_t

(...)

(∘ ∘ ∘)

KCL @ c:

$$\frac{V_c - V_d}{R_3} + \frac{V_c - V_t}{R_5} + \frac{V_c}{R_4} = 0$$

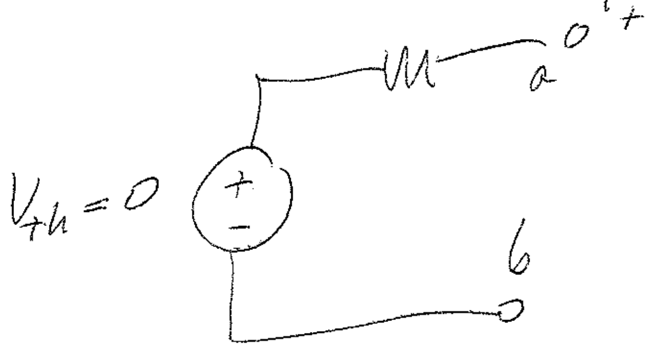
KCL @ d:

$$\frac{V_d - V_T}{R_1} + \frac{V_d - V_c}{R_3} + \frac{V_d}{R_2} = 0$$

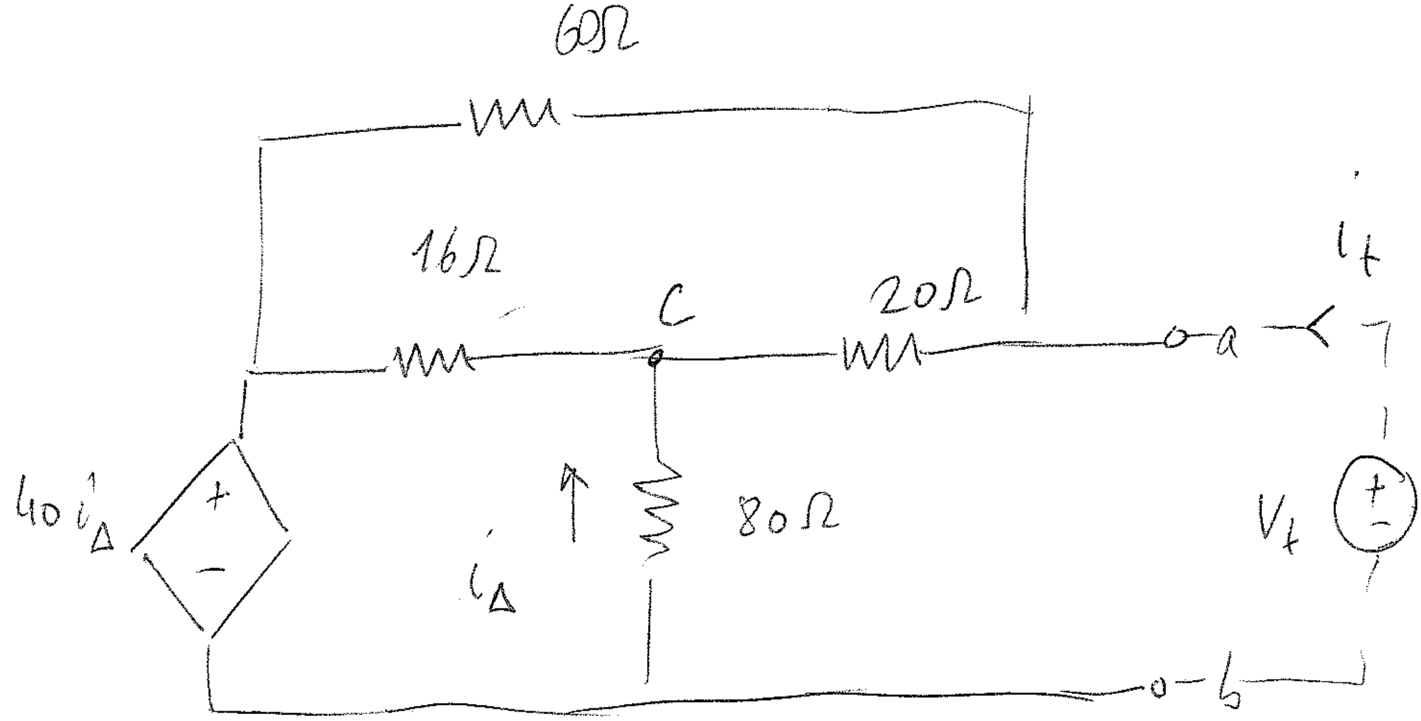
now give V_T a value like 1V &
solve for V_c & V_d.

$$i_t = -\left(\frac{V_c - V_t}{R_5} + \frac{V_d - V_t}{R_1}\right) \quad \left[\begin{array}{c} \text{KCL} \\ @\ a \end{array}\right]$$

$$R_{Th} = \frac{V_t}{i_t}$$



Slide 34 :



$V_{oc} = 0$, $I_{sc} = 0$ $R_{th} = \dfrac{V_{oc}}{I_{sc}} = ?$

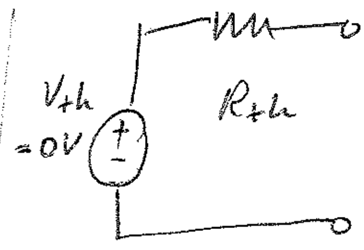
KCL @ C

2 eq
2 unknown

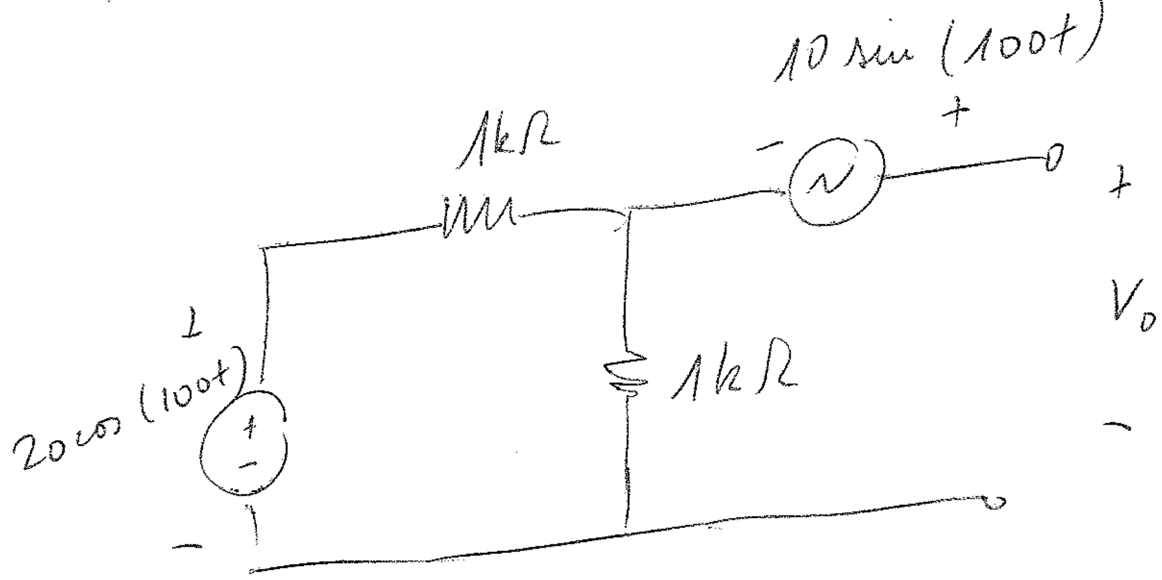
$$\begin{cases} \dfrac{V_c - 40 i_\Delta}{16} + \dfrac{V_c}{80} + \dfrac{V_c - V_T}{20\Omega} = 0 \\[2ex] i_\Delta = -\dfrac{V_c}{80\Omega} \end{cases}$$

Solve for V_c by making V_t like 1 V.

Solve for i_t & $R_{th} = \dfrac{V_t}{i_t}$



Slide 35

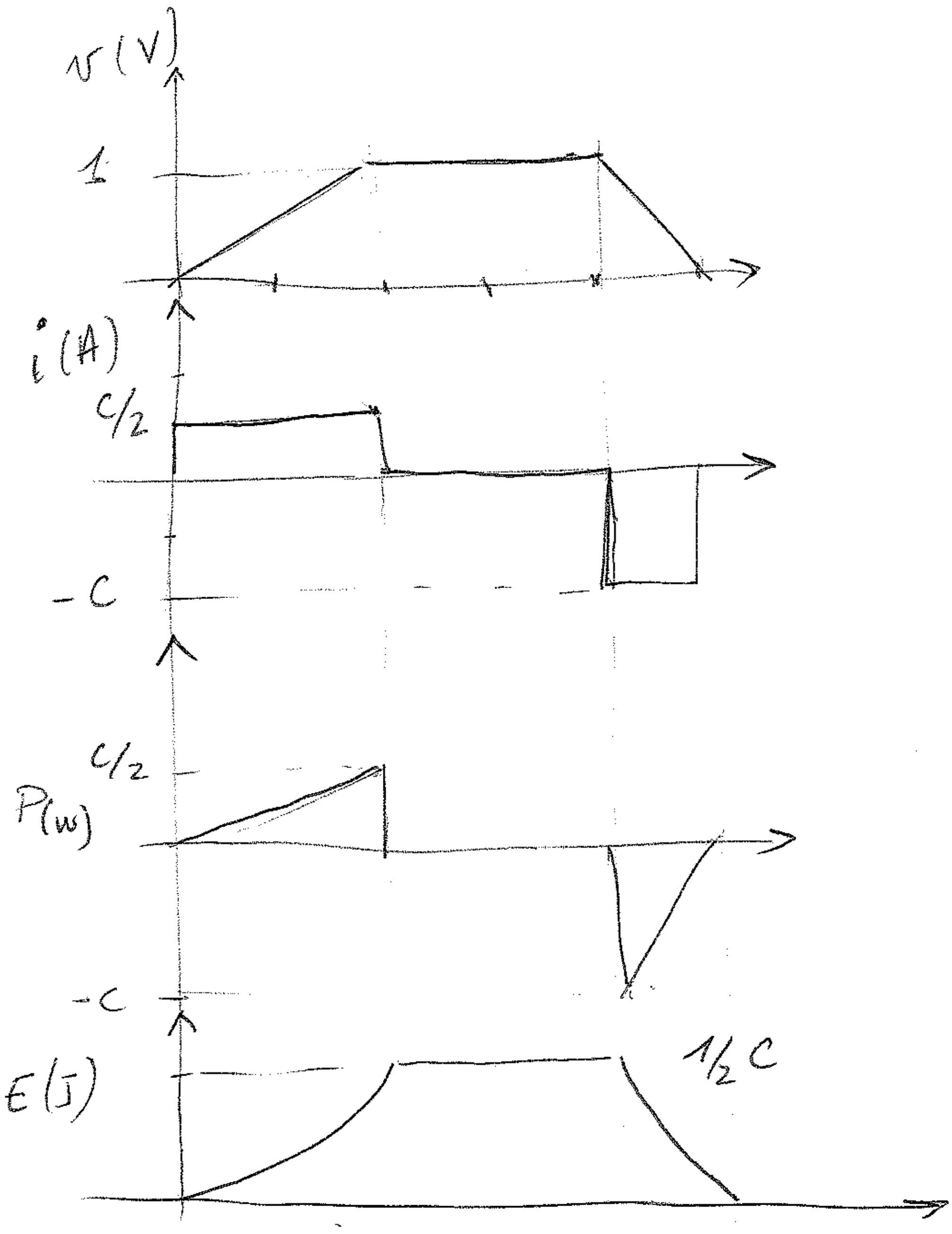


By superposition: $V_0 = 10\cos(100t) + 10\sin(100t)$

Lecture 4

Slide 3: previously done in another lecture.

Slide 10:



Slide 12

$$V_1(t) = \frac{1}{C_1}\int i(t)\,dt + \frac{Q_1}{C_1}$$

$$V_2(t) = \frac{1}{C_2}\int i(t)\,dt + \frac{Q_2}{C_1}$$

$$V(t) = V_1(t) + V_2(t) = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int i(t)\,dt + \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

V(t) also equals:

$$V(t) = \frac{1}{C_{eq}}\int i(t) + \frac{Q_{eq}}{C_{eq}}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$Q_{eq} = C_{eq}\left[\frac{Q_1}{C_1} + \frac{Q_2}{C_2}\right]$$

Slide 13

$$i_1(t) = C_1 \frac{dv(t)}{dt}$$

$$i_2(t) = C_2 \frac{dv(t)}{dt}$$

$$i(t) = i_1(t) + i_2(t) = (C_1 + C_2) \frac{dv(t)}{dt}$$

$i(t)$ also equals: $i(t) = C_{eq} \frac{d(v(t))}{dt}$

$$\Rightarrow \underline{C_{eq} = C_1 + C_2}$$

Slide 16:

$$p(t_0) = v(t_0)\, i(t_0) = L \frac{di(t_0)}{dt} \cdot i(t_0)$$

$$= L \frac{di(t_0)}{dt} \cdot i_0$$

$$w(t) = \int_{t_0}^{t} p(\tau)\, d\tau = \int_{t_0}^{t} v(\tau)\, i(\tau)\, d\tau = \int_{t_0}^{t} L \frac{di(\tau)}{d(\tau)}\, i(\tau)\, d\tau$$

$$= \int_{t_0}^{t} L i(\tau)\, d\tau = \frac{1}{2} L i^2 - \frac{1}{2} L i_0^2$$

Slide 17:

$$V_1(t) = L_1 \frac{di(t)}{dt}$$

$$V_2(t) = L_2 \frac{di(t)}{dt}$$

$$v(t) = V_1(t) + V_2(t) = (L_1 + L_2) \frac{di(t)}{dt}$$

$v(t)$ also equals:

$$v(t) = L_{eq} \frac{di(t)}{dt} \Rightarrow L_{eq} = L_1 + L_2$$

Slide 18:

$$i_1(t) = \frac{1}{L_1} \int v(t)dt + i_1(0)$$

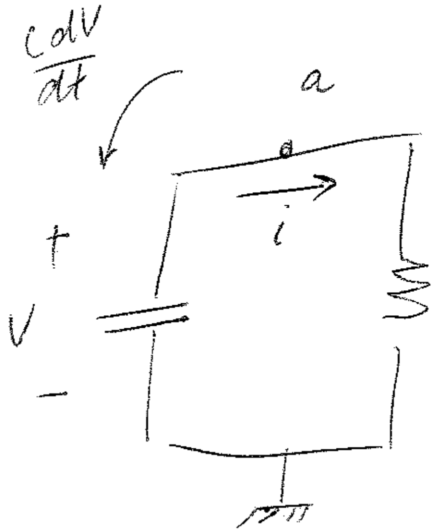
$$i_2(t) = \frac{1}{L_2} \int v(t)dt + i_2(0)$$

$$i(t) = i_1(t) + i_2(t) = \left[\frac{1}{L_1} + \frac{1}{L_2}\right] \int v(t)dt + i_1(0) + i_2(0)$$

$i(t)$ also equals:

$$i(t) = \frac{1}{L_{eq}} \int v(t)dt + i_{eq}(0) \Rightarrow \begin{cases} \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \\ i_{eq}(0) = i_1(0) + i_2(0) \end{cases}$$

Slide 29:



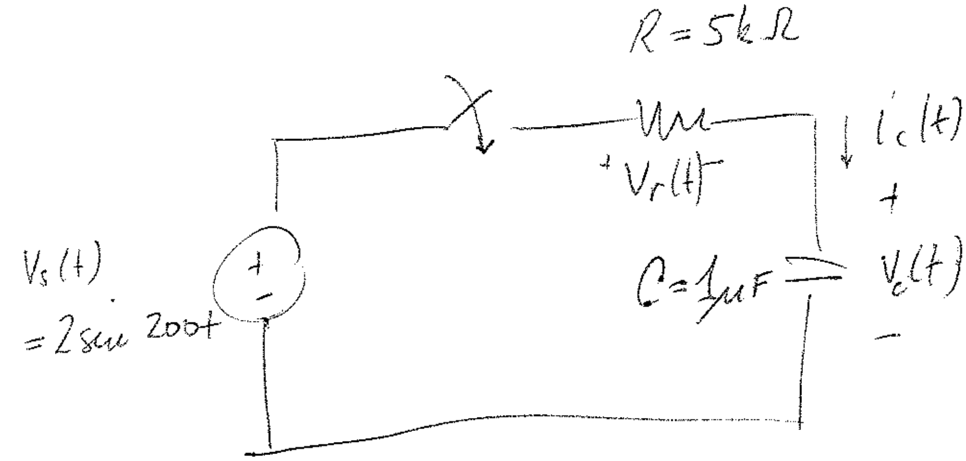
KCL :

$$+\frac{CdV}{dt} + V/R = 0$$

$$\Rightarrow V(t) + RC\frac{dV(t)}{dt} = 0$$

Lecture 5:

slide 9:



KVL:

$$-V_s(t) + V_R(t) + V_c(t) = 0$$

$$RC\frac{dV_c(t)}{dt} + V_c(t) = V_s(t)$$

Complementary solution: Ke^{st} for $RC\frac{dV_c(t)}{dt} + V_c(t) = 0$ yields $K=0$ since the DC value of $V_c(t)$ stays at 0V.

Particular Solution:

$$(1)\quad RC\frac{dV_c(t)}{dt} + V_c(t) = 2\sin 200t$$

$V_c(t)$ is of the form: $A\cos 200t + B\sin 200t$

replace back into 1:

$$RC\cdot\left[-200A\sin 200t + B\,200\cos 200t\right]$$

$$+ A\cos 200t + B\sin 200t = 2\sin 200t$$

$$\Rightarrow \begin{cases} -200ARC + B = 2 \\ RC\cdot B\cdot 200 + A = 0 \end{cases}$$

solve for A & B

Lecture 6

Slide 24:

$$v(t) = 120V \cos(377t + 30°)$$

$$V = 120 \angle 30°$$

$$I = \frac{V}{Z_c} \qquad Z_c = \frac{1}{j\omega C}$$

$$\Rightarrow I = j\omega C \cdot V$$

$$= \omega C \cdot |V| \angle(\angle V + 90°)$$

$$\Rightarrow i(t) = \omega C \cdot 120V \cos(377t + 120)$$

where $\omega = 377 \frac{rad}{s}$

Slide 26:

$$i(t) = 1\mu A \cos(2\pi \cdot 9.15 \cdot 10^7 t + 30°)$$

$$I = 1\mu \angle 30°$$

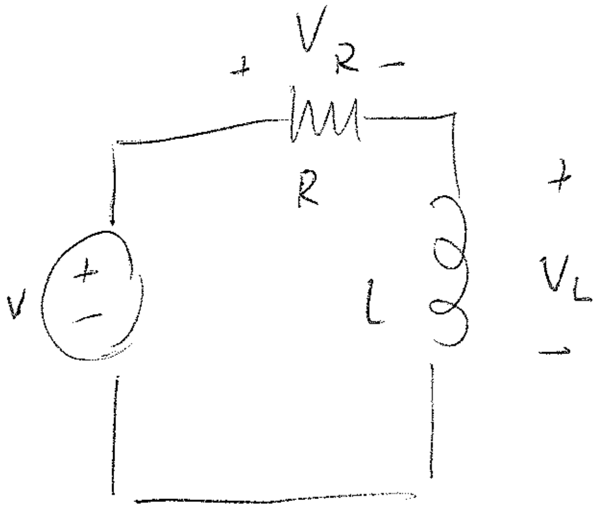
$$Z_L = j\omega L$$

$$V = I \cdot Z = j\omega L \cdot 1\mu \angle 30$$

$$= \omega \cdot L \cdot 1\mu \angle 30 + 90$$

$$v(t) = \omega L \cdot 1\mu A \cos(\omega t + 120°)$$

Lecture 7:



1st Order Lowpass filter:

$$H(j\omega) = \frac{V_R}{V} = \frac{R}{R + j\omega L}$$

$$H(j\omega) = \frac{1}{1 + j\omega \frac{L}{R}} = \frac{1}{1 + j\frac{\omega}{\omega_B}}$$

where $\omega_B = \frac{R}{L}$

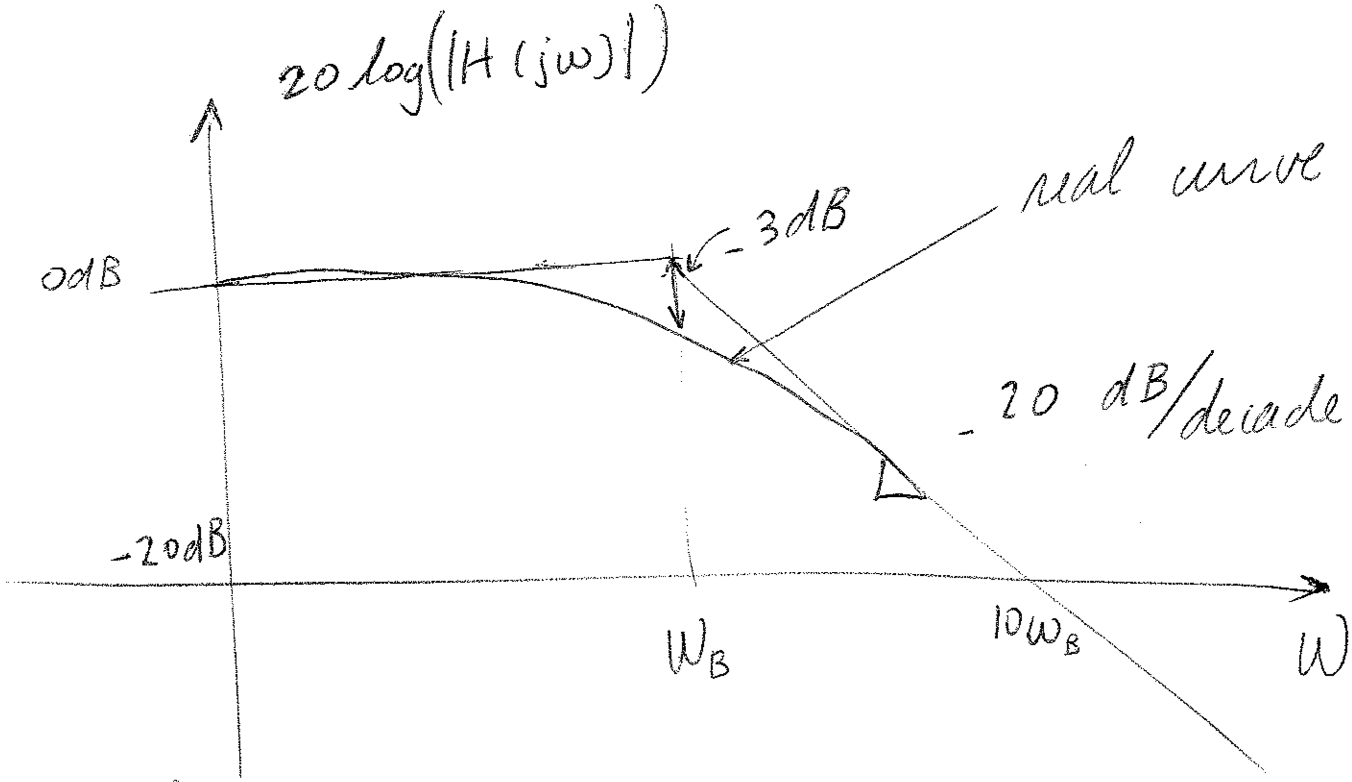
1st Order highpass filter:

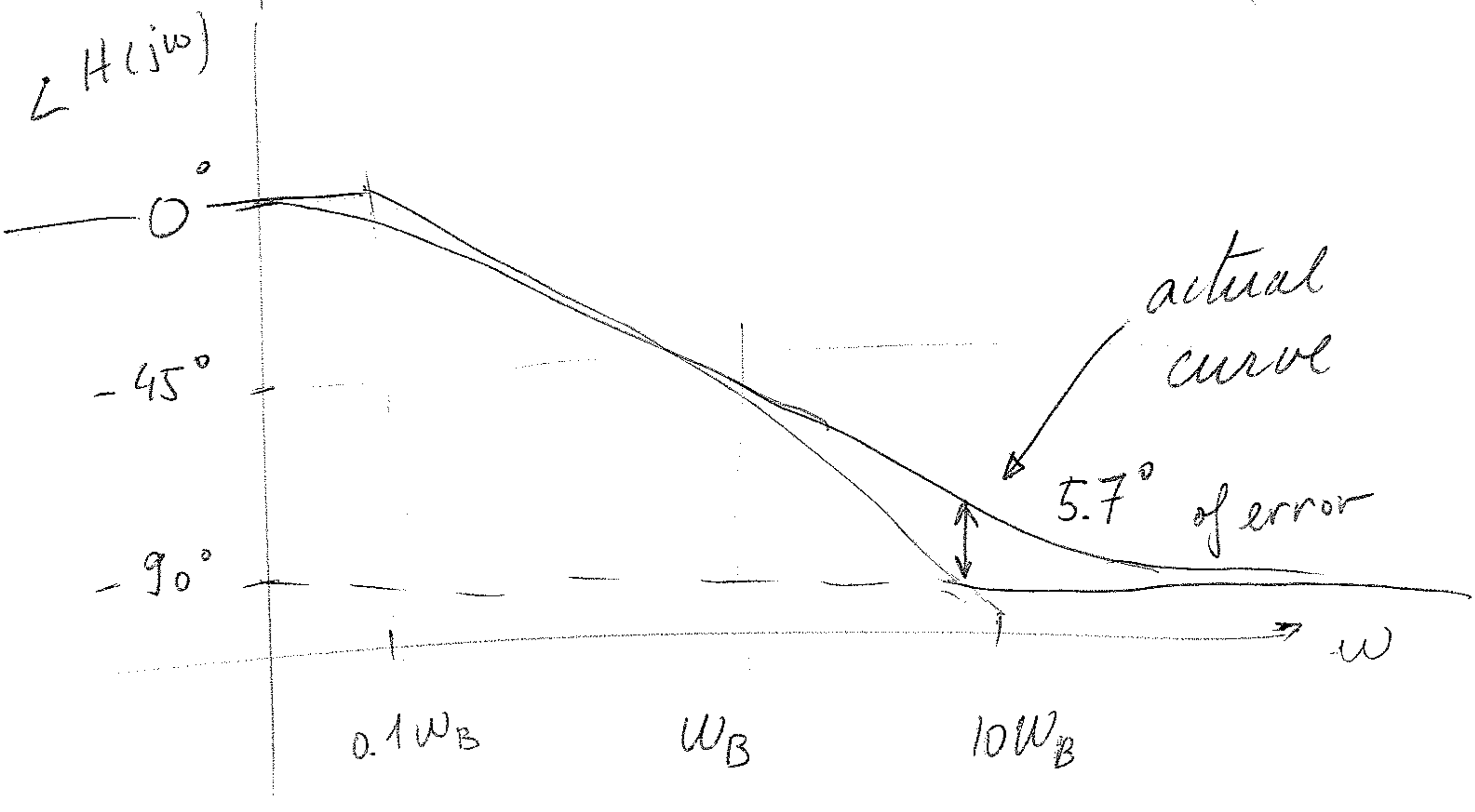
$$H(j\omega) = \frac{V_L}{V} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega L/R}{1 + j\omega L/R}$$

$$H(j\omega) = \frac{j\,\omega/\omega_B}{1 + j\,\omega/\omega_B}$$ where $\omega_B = R/L$

Bode Plot:

$$H(j\omega) = \frac{1}{1+j\,\omega/\omega_B} = \sqrt{\frac{1}{1+\left(\frac{\omega}{\omega_B}\right)^2}}\;\angle -\tan^{-1}\frac{\omega}{\omega_B}$$





Bode plot of high pass filter:

$$H(j\omega) = \frac{j\,\omega/\omega_B}{1 + j\,\omega/\omega_B}$$

$$H(j\omega) = \frac{\omega/\omega_B}{\sqrt{1+\left(\frac{\omega}{\omega_B}\right)^2}} \angle \pi/2 - \tan^{-1}\left(\frac{\omega}{\omega_B}\right)$$

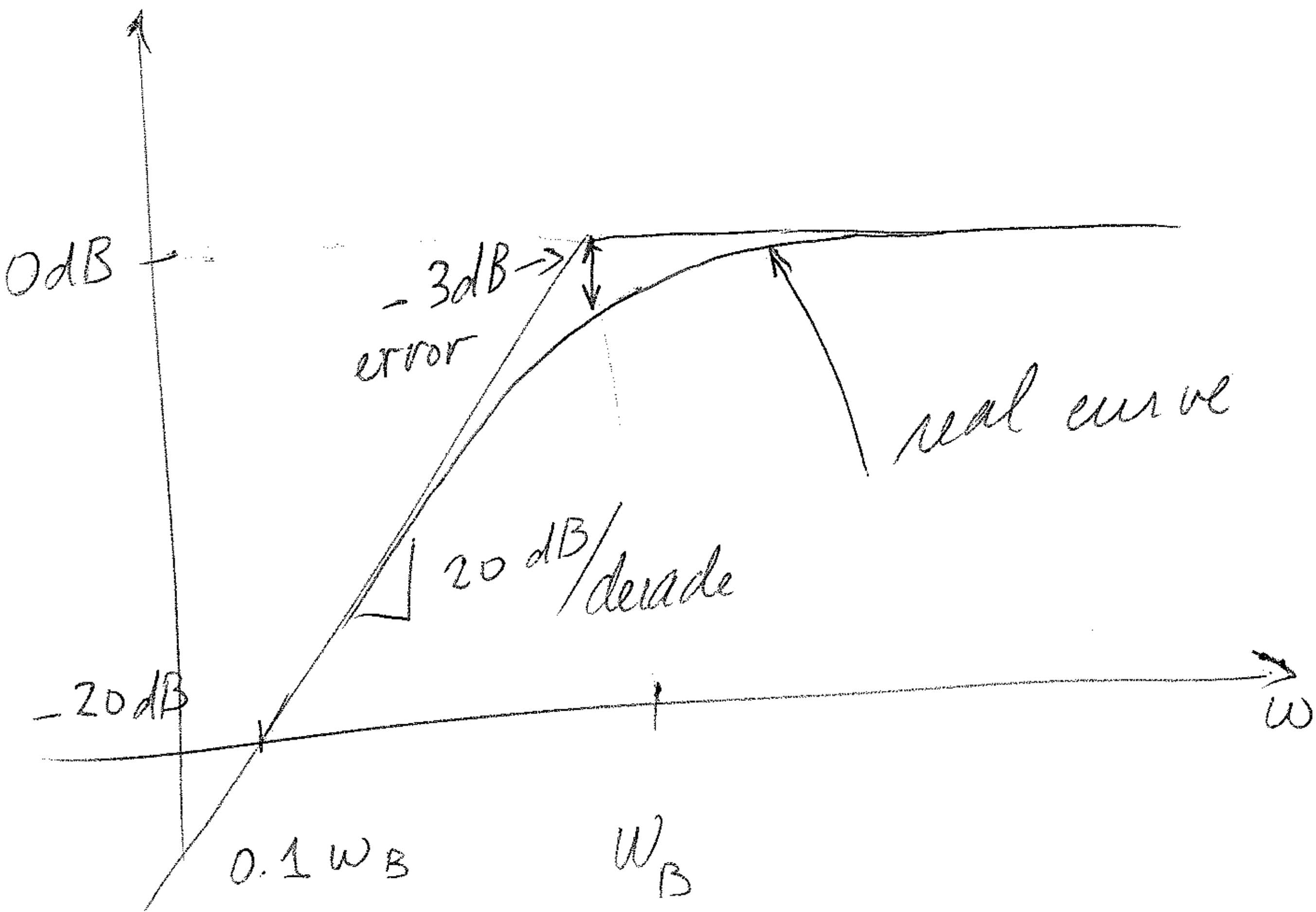
Magnitude plot:

$$|H(j\omega_B)| = \frac{1}{\sqrt{2}} \equiv -3\,dB$$

for $\omega >> \omega_B$, $|H(j\omega)| \simeq 1 \equiv 0\,dB$

for $\omega << \omega_B$, $|H(j\omega)| \simeq \frac{\omega}{\omega_B} \equiv$ slope of $+20\,dB/\text{decade}$

Magnitude Plot of highpass filter:

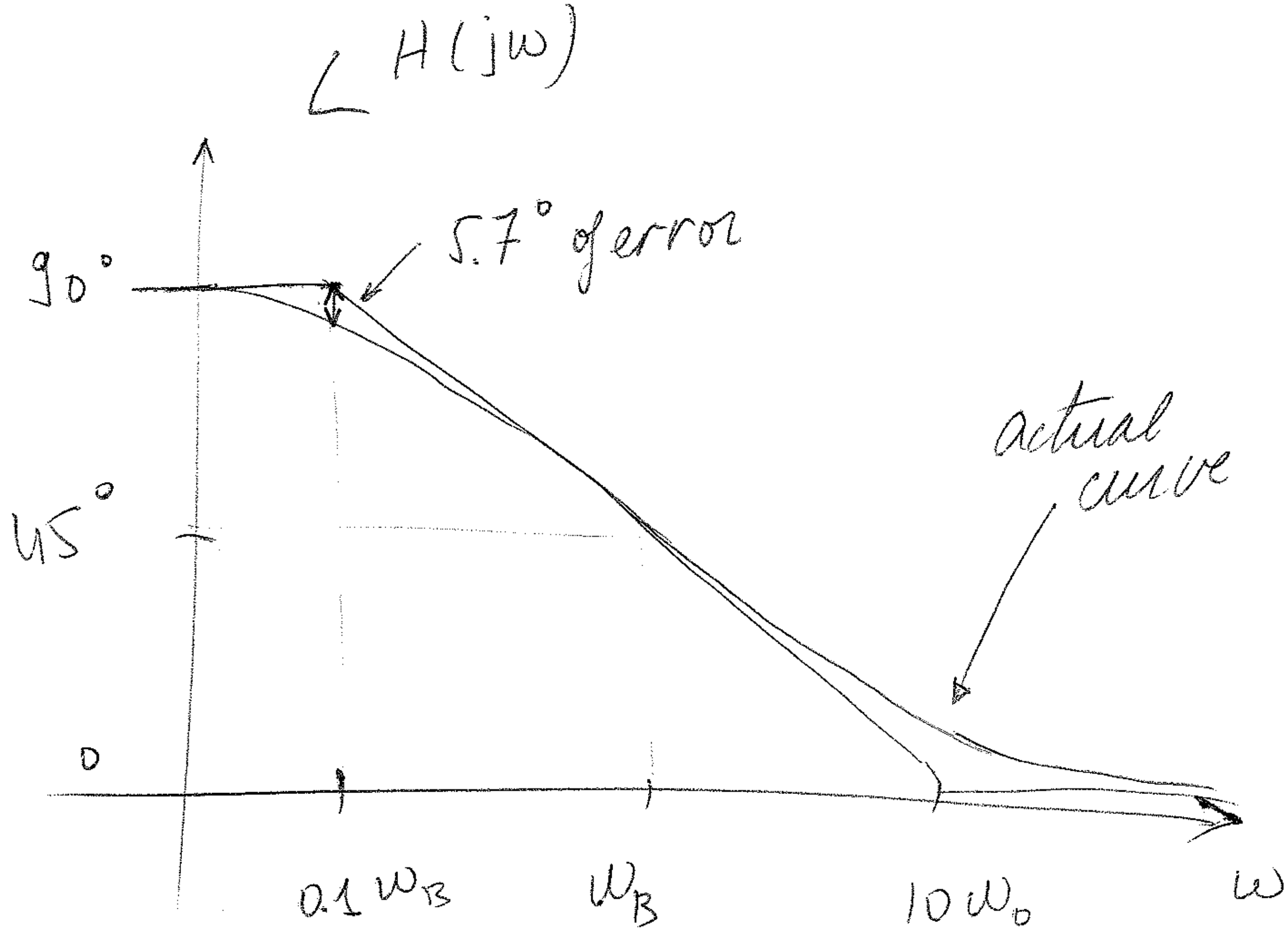


Phase plot of high pass filter.

$$\angle H(j\omega_B) = 45^\circ$$

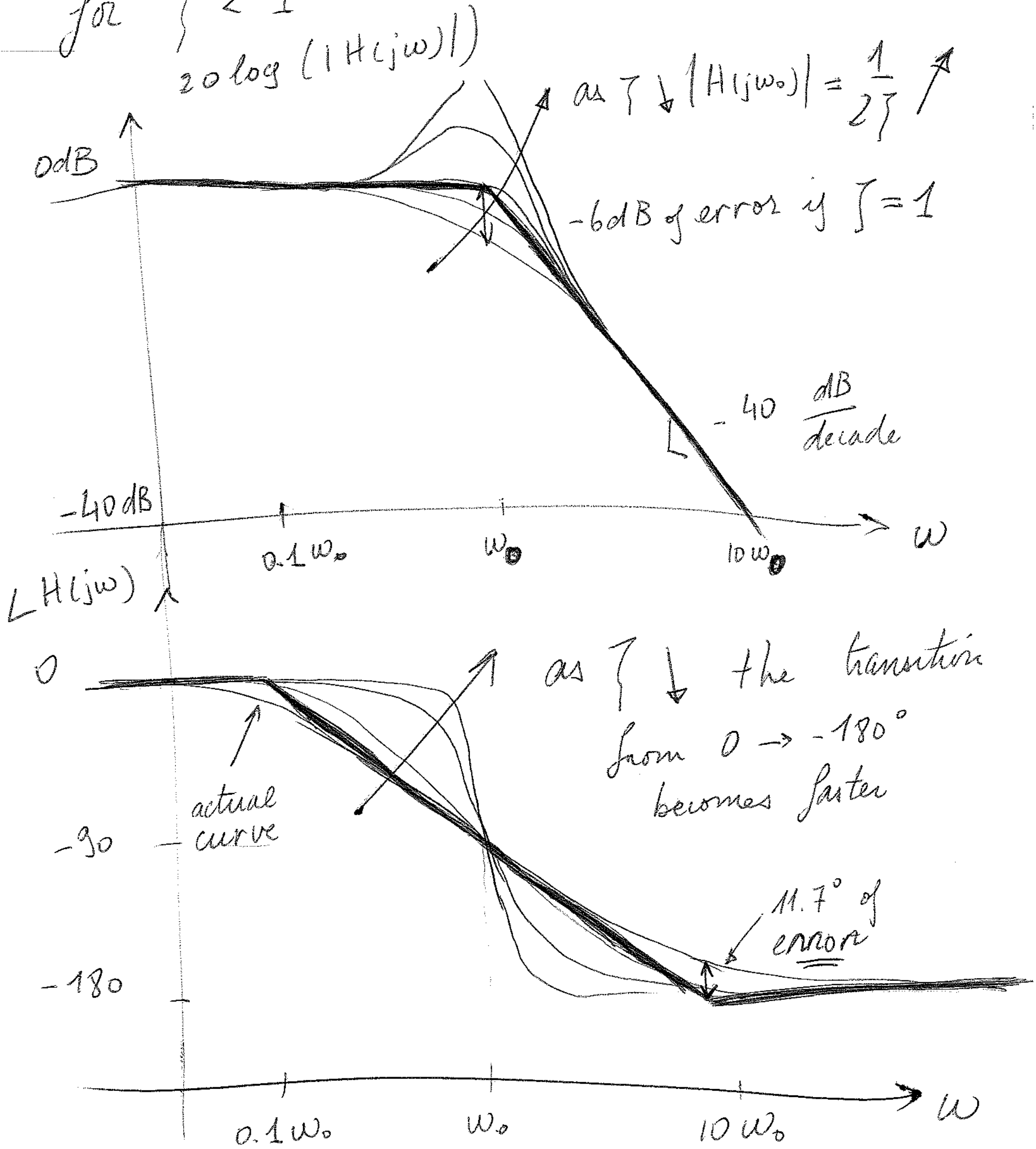
$$\angle H(j\,0.1\,\omega_B) = 84.3^\circ \approx 90^\circ$$

$$\angle H(j\,10\,\omega_B) = 5.7^\circ \approx 0^\circ$$



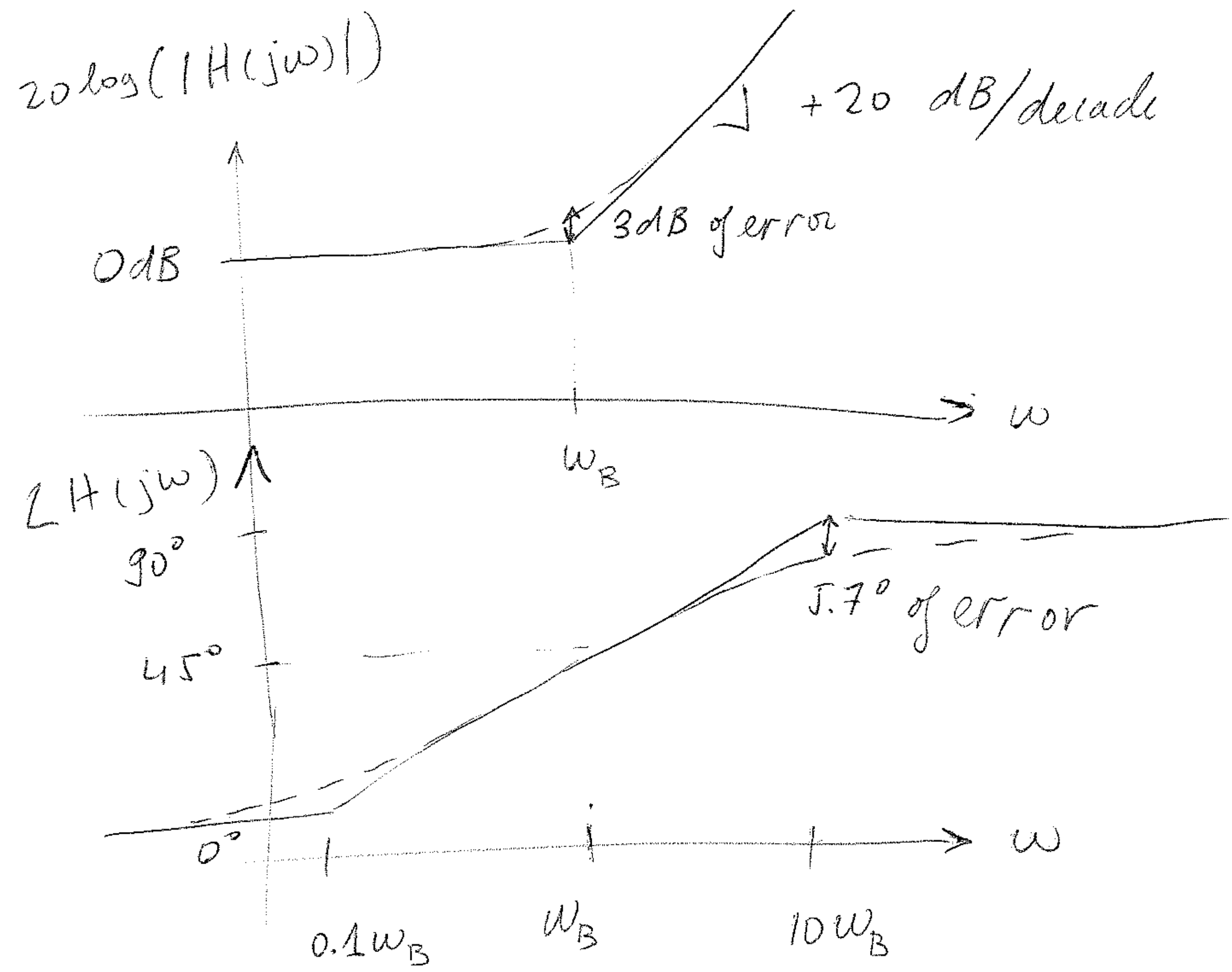
Bode plot of second order low pass filter.

for $\zeta < 1$

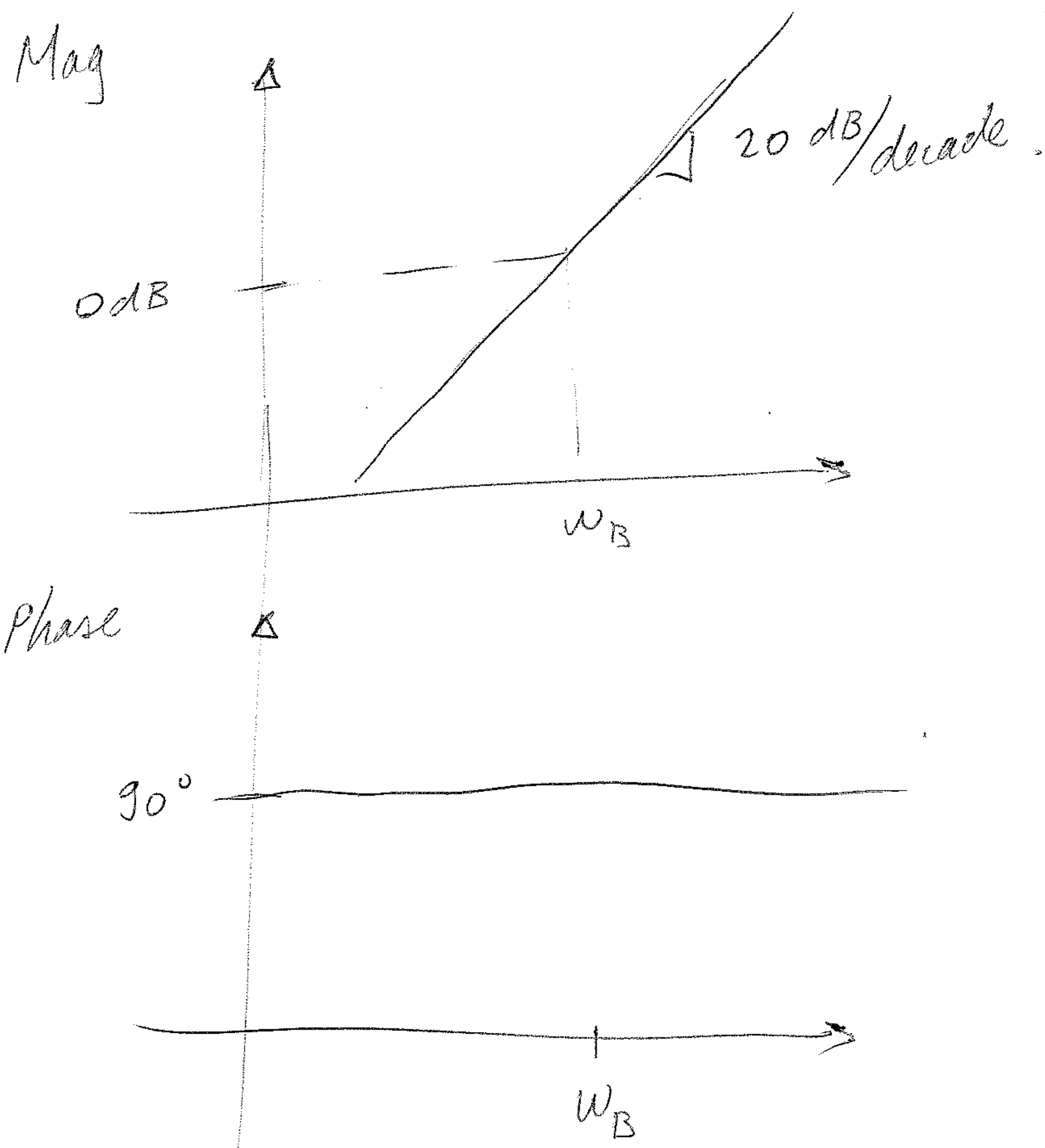


Higher Order Bode plots:

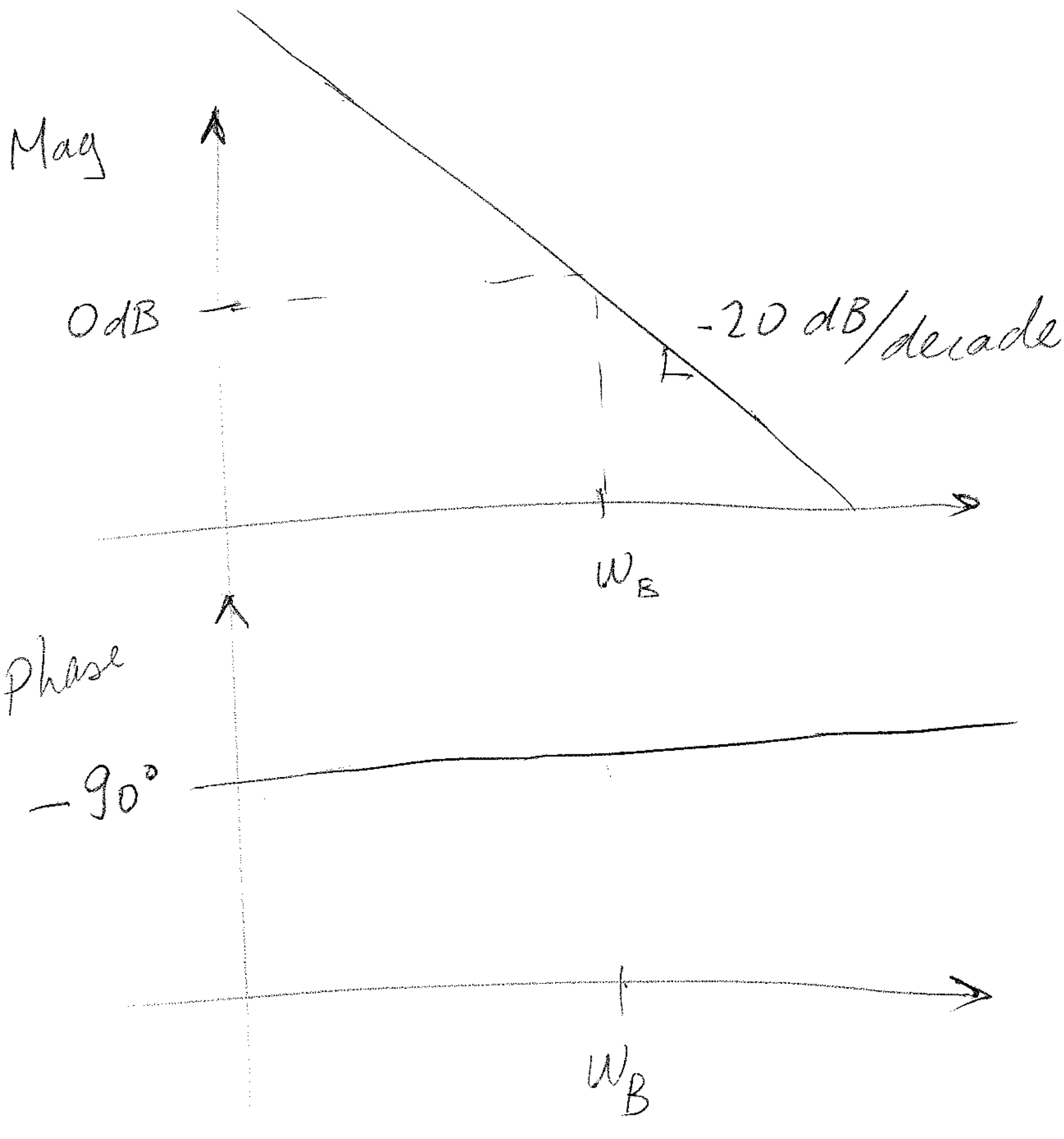
For: $H(j\omega) = 1 + \dfrac{j\omega}{\omega_B}$

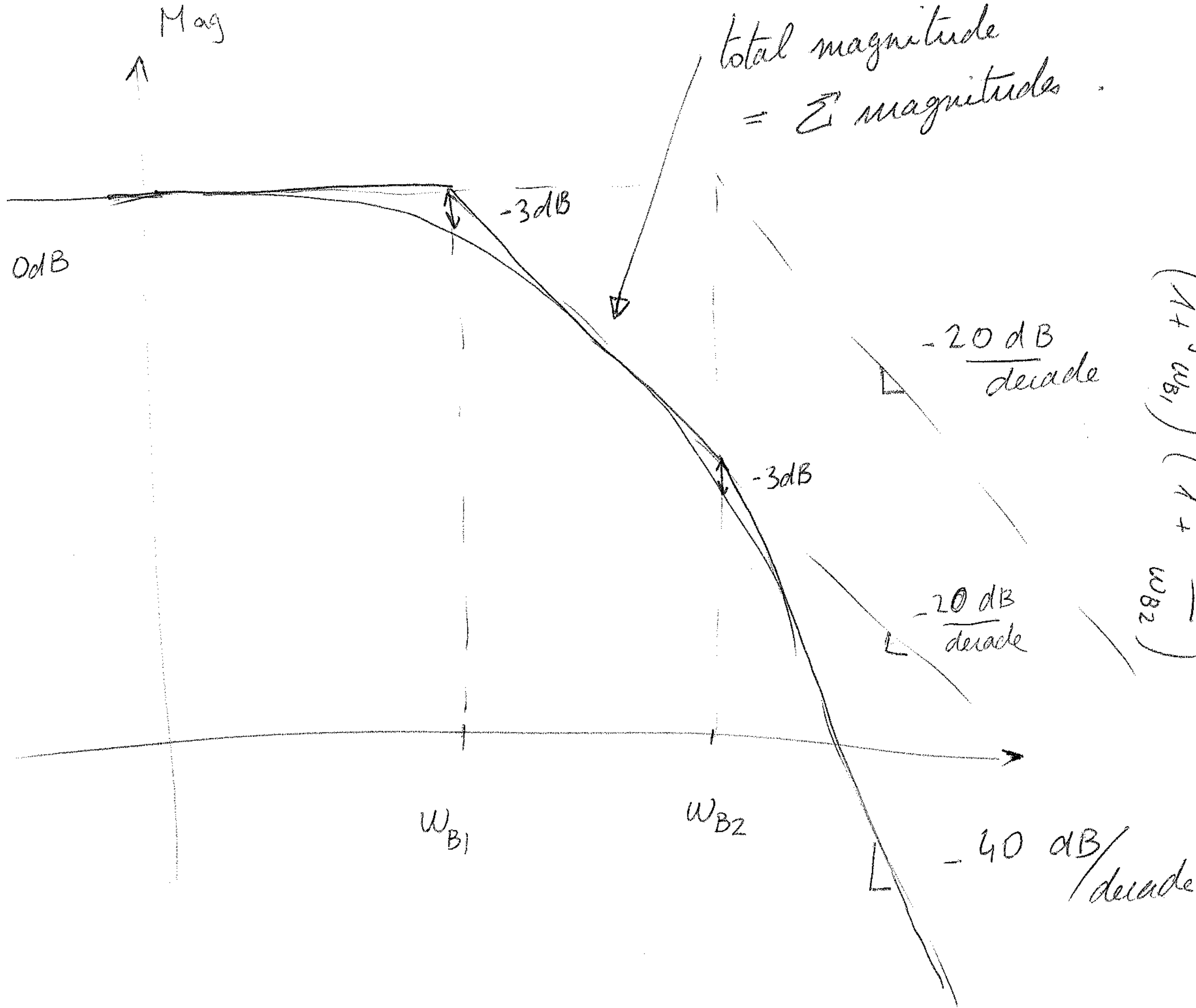


For $H(j\omega) = j\omega/\omega_B$

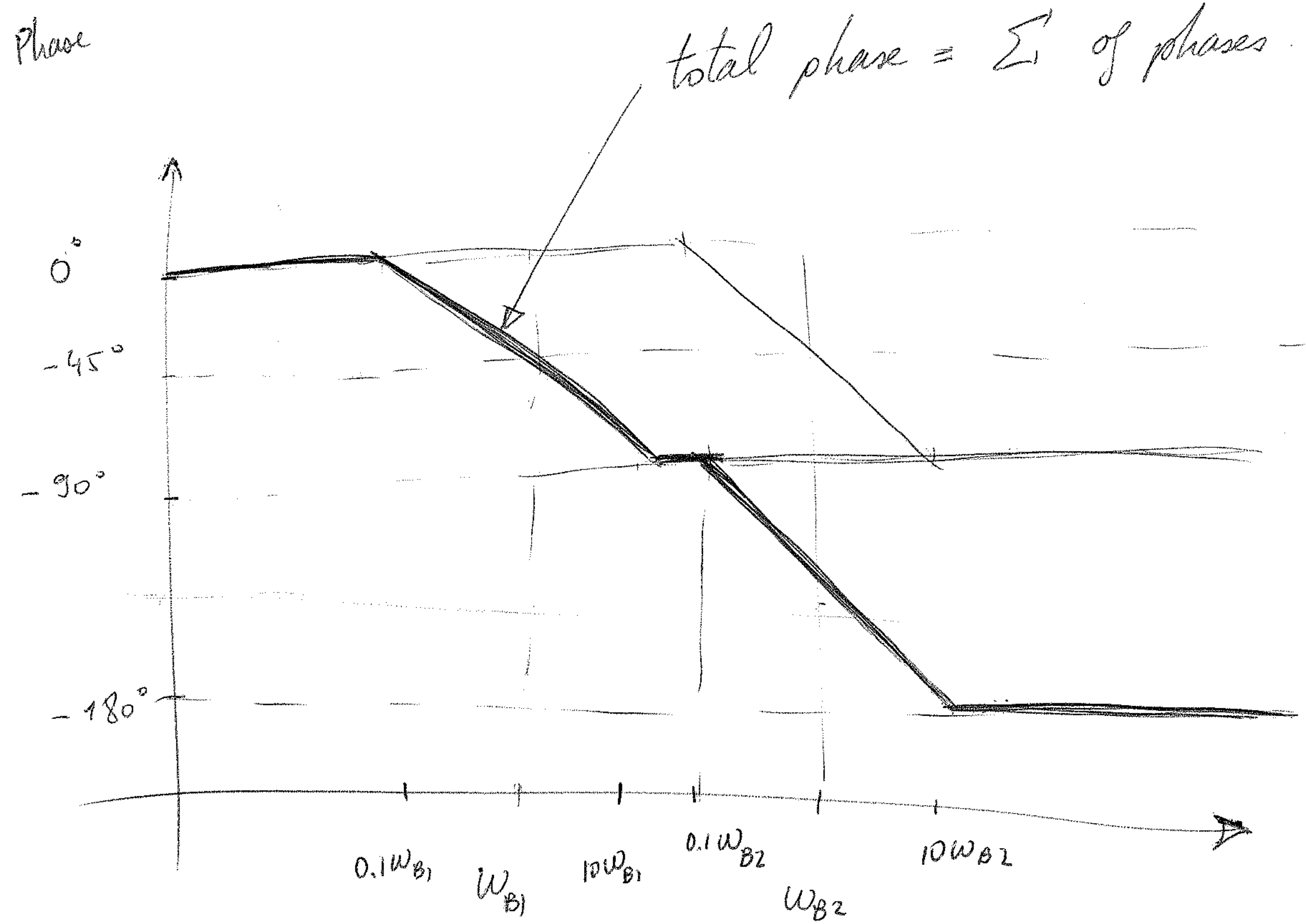


For $H(j\omega) = \dfrac{1}{j\,\omega/\omega_B}$

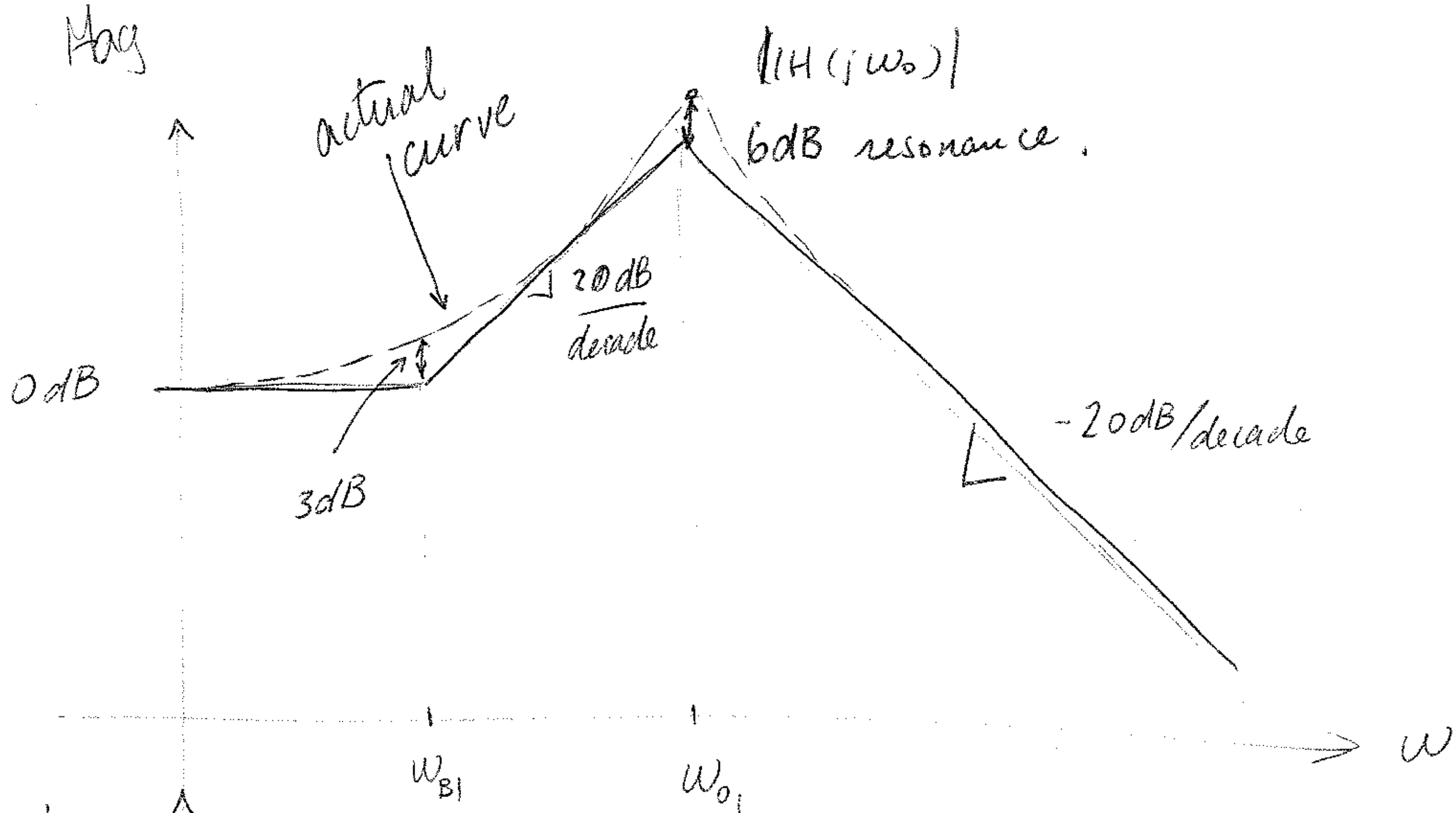


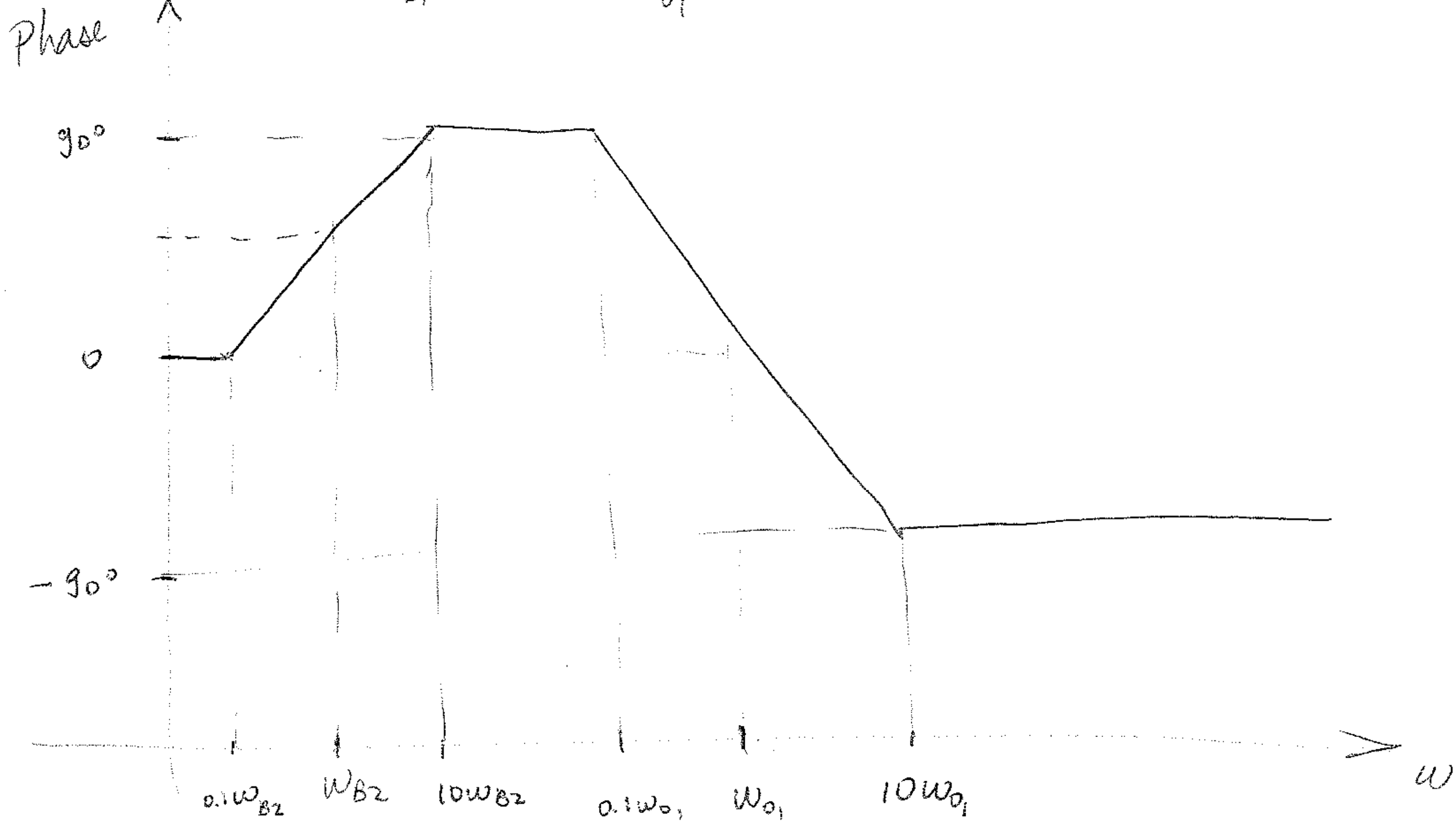


$$H(j\omega) = \frac{1}{\left(1 + \frac{j\omega}{\omega_{B1}}\right)\left(1 + \frac{j\omega}{\omega_{B2}}\right)}$$


Phase
total phase = Σ of phases
0°
-45°
-90°
-180°
0.1ω_B1
ω_B1
10ω_B1
0.1ω_B2
ω_B2
10ω_B2

$$H(j\omega) = \frac{\left(1 + \frac{j\omega}{\omega_B}\right)}{1 + 2\zeta\frac{j\omega}{\omega_0} + \left(\frac{j\omega}{\omega_0}\right)^2} \qquad \zeta = 1/4$$





Lecture 8:

Slide 3:

$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1} + L_3$$

$$Z_{eq} = \left(\frac{1}{j\omega L_1} + \frac{1}{j\omega L_2}\right)^{-1} + j\omega L_3$$

slide 4:

$$C_{eq} = \left(\frac{1}{C_1 + C_2} + \frac{1}{C_3}\right)^{-1}$$

$$Z_{eq} = \frac{1}{j\omega C_3} + \left[j\omega C_1 + j\omega C_2\right]^{-1}$$

$$\cancel{\Delta V_0 = \Delta V_i}$$

$$\Delta Q = \Delta V_i \cdot C_{eq}$$

$$\Delta V_0 = \frac{\Delta Q}{C_3} = \Delta V_i \cdot \frac{C_{eq}}{C_3}$$

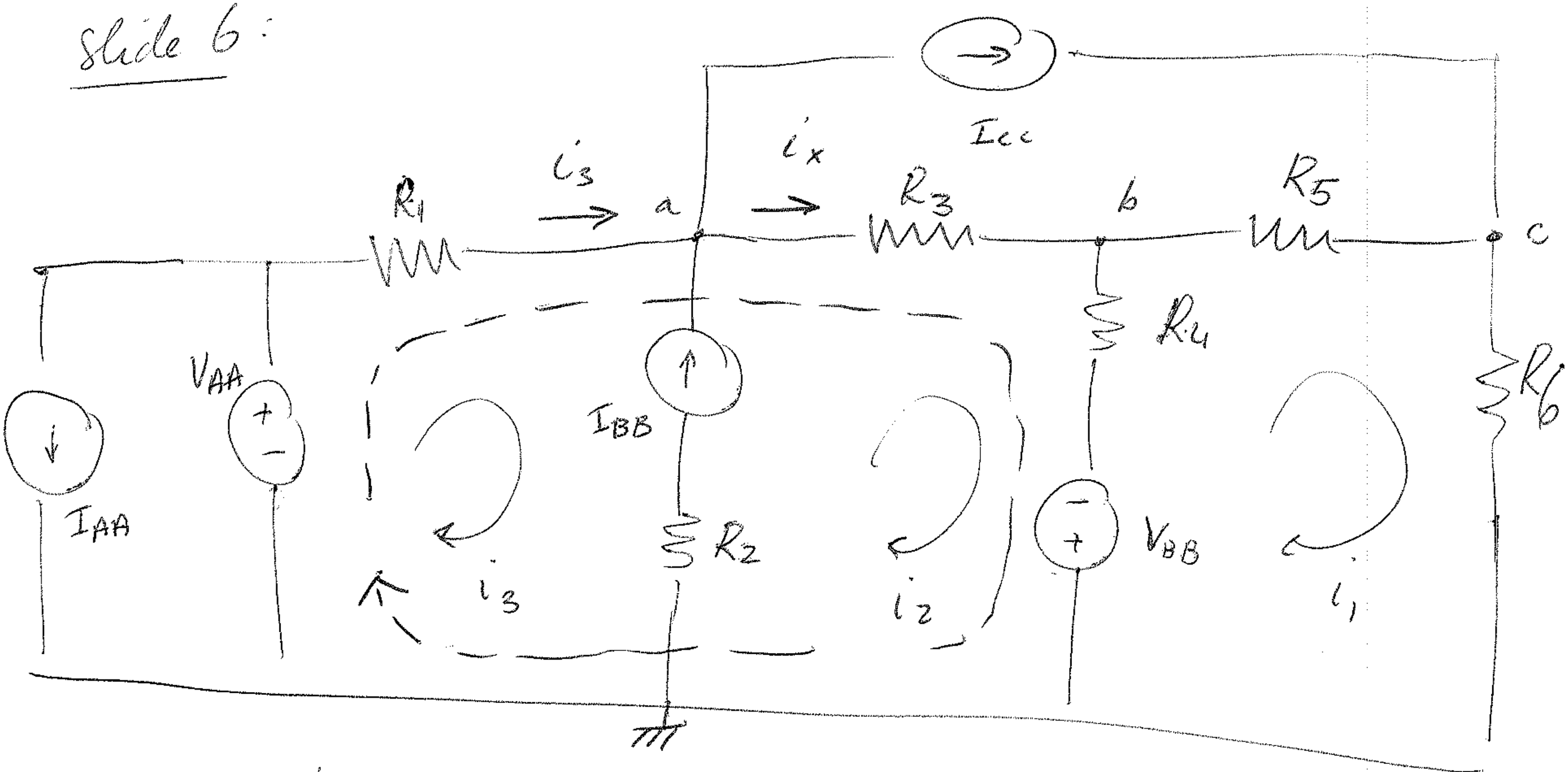
slide 5:

$$Q_1 = C_1 \cdot V_1 \qquad Q_2 = C_2 \cdot V_2$$

$$Q_{total} = Q_1 + Q_2.$$

$$V_{final} = \frac{Q_{total}}{C_{eq}} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Slide 6:



KVL @ i_1: $V_{BB} + (i_1 - i_2)R_4 + (i_1 - I_{cc})R_5 + i_1 R_6 = 0V$

KVL @ supermesh:

$$-V_{AA} + i_3 R_1 + (i_2 - I_{cc}) R_3 + (i_2 - i_1) R_4 + (-V_{BB}) = 0V$$

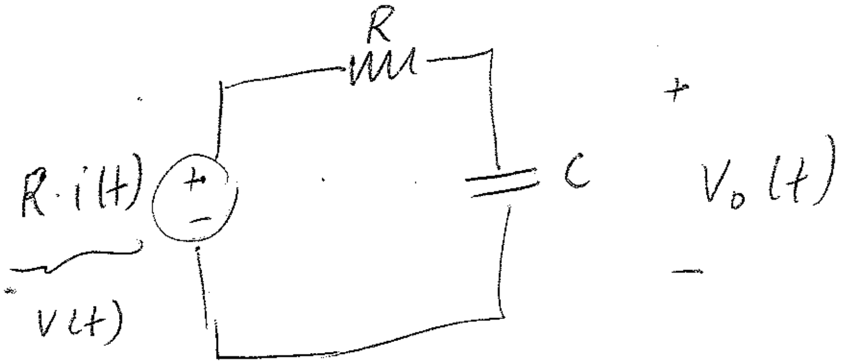
we need one more equation

KCL @ a: $-i_3 + I_{cc} + i_x - I_{BB} = 0$ where $i_x = i_2 - I_{cc}$

$$\Rightarrow I_{BB} = i_2 - i_3$$

Slide 7 :

Using a source transformation, the circuit becomes:



$v(t) = 2 + 3\mu(t)$.

Solution 1 :

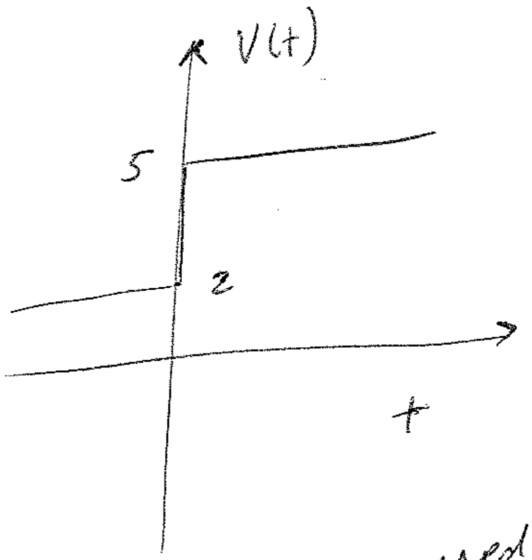
KVL : $-v(t) + R \cdot i(t) + V_o(t) = 0$

$i(t) = \frac{C\, dV_o(t)}{dt}$

$\Rightarrow \quad V_o(t) + RC \frac{dV_o(t)}{dt} = v(t)$

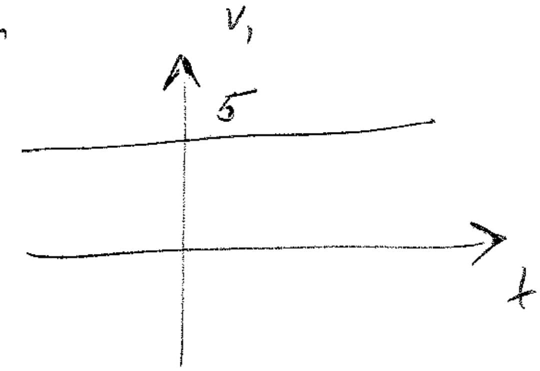
$$V_o(t) = \underbrace{V_c(t)}_{\text{complementary solution}} + \underbrace{V_p(t)}_{\text{particular solution}}$$

$v(t) = 2 + 3u(t)$



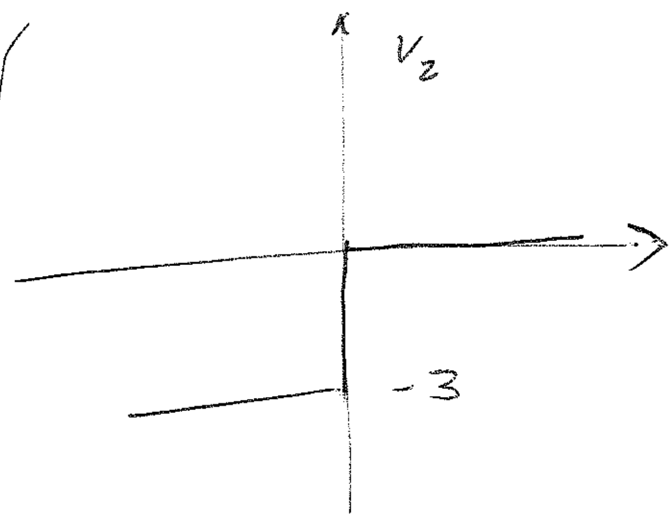
=

used to find $V_p(t)$



+

used to find $V_c(t)$



for $t \geq 0$

$$V_c(t) + RC \frac{dV_c(t)}{dt} = 0$$

$$V_c(t) = Ke^{-t/\tau}$$

we know that $V_c(0) = -3V$

$$\Rightarrow V_c(t) = -3e^{-t/\tau} \quad \text{where } \tau = RC$$

also we know $V_p(t) = 5$

$$\Rightarrow V_o(t) = V_c(t) + V_p(t) = 5 - 3e^{-t/\tau}$$

Solution 2:

$$V_o(t) + RC \frac{dV_o(t)}{dt} = v(t)$$

$$V_o(t) = K_1 + K_2 e^{-t/\tau}$$

Initial conditions:

$V_o(0) = 2V \Rightarrow K_1 + K_2 = 2$

$\lim_{t \to \infty} v_o(t) = 5V \Rightarrow K_1 = 5V$

$$\Rightarrow V_o(t) = 5 - 3e^{-t/\tau}$$

Solution 3:

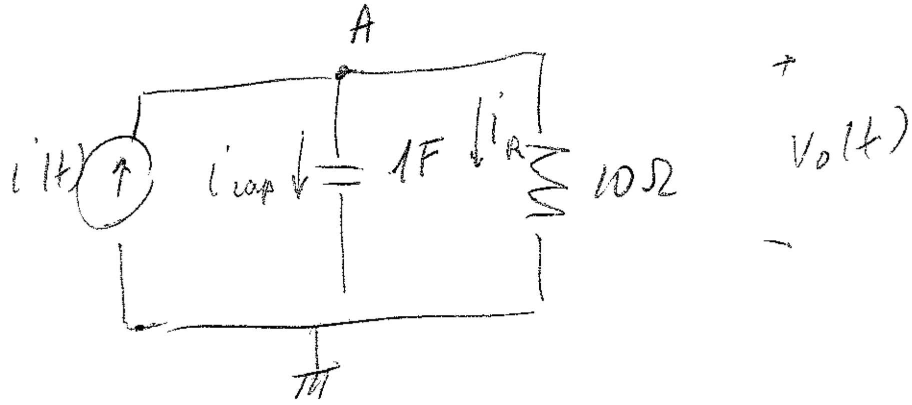
$$v_o(t) = V_f - (V_f - V_i)e^{-t/\tau}$$

$V_f = 5V \qquad V_i = 2V$

$$\Rightarrow V_o(t) = 5 - 3e^{-t/\tau} \quad \text{where } \tau = RC$$

Slide 7:

1^{st} Order Circuit:



KCL @ A: $\quad -i(t) + i_{cap}(t) + i_R = 0$

$$C\frac{d\,V_o(t)}{dt} + \frac{V_o(t)}{R} = i(t)$$

v(t) from previous example.

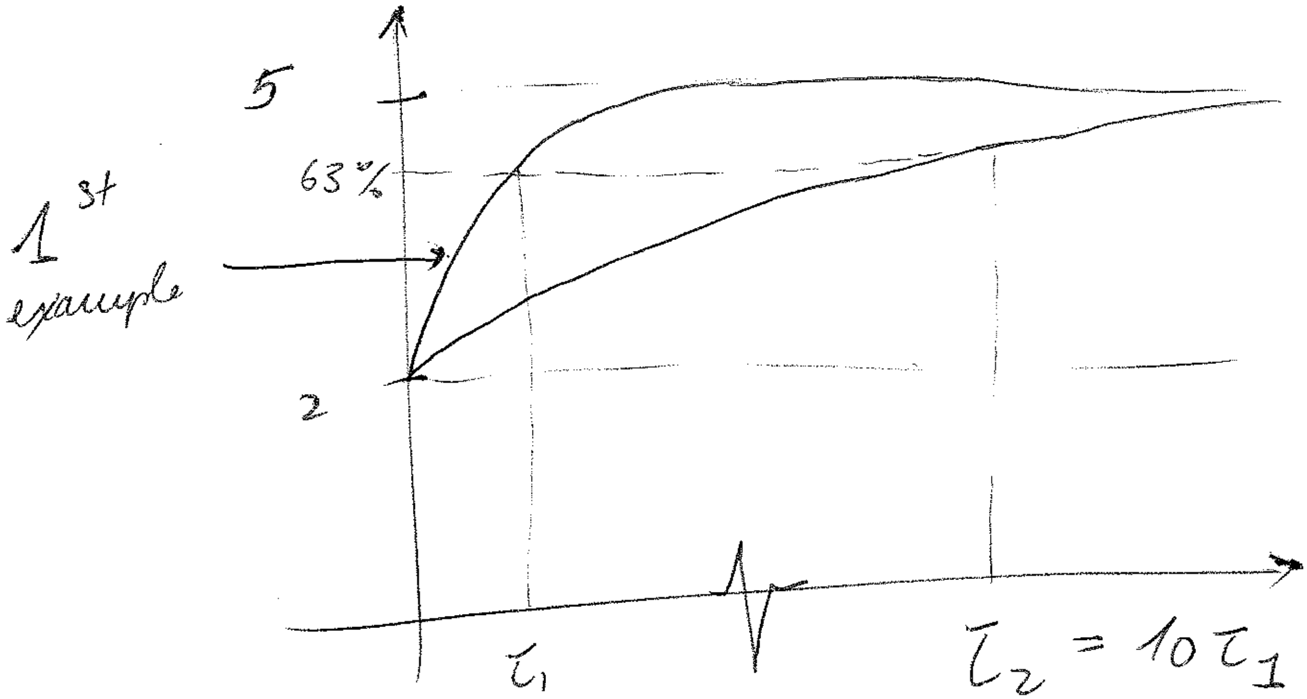
$$\underbrace{\Rightarrow V_o(t) + RC\frac{dV_o(t)}{dt} = \overbrace{R\,i(t)}}_{}$$

Note: same equation as when we did a source transformation.

$\Rightarrow$ we know the solution

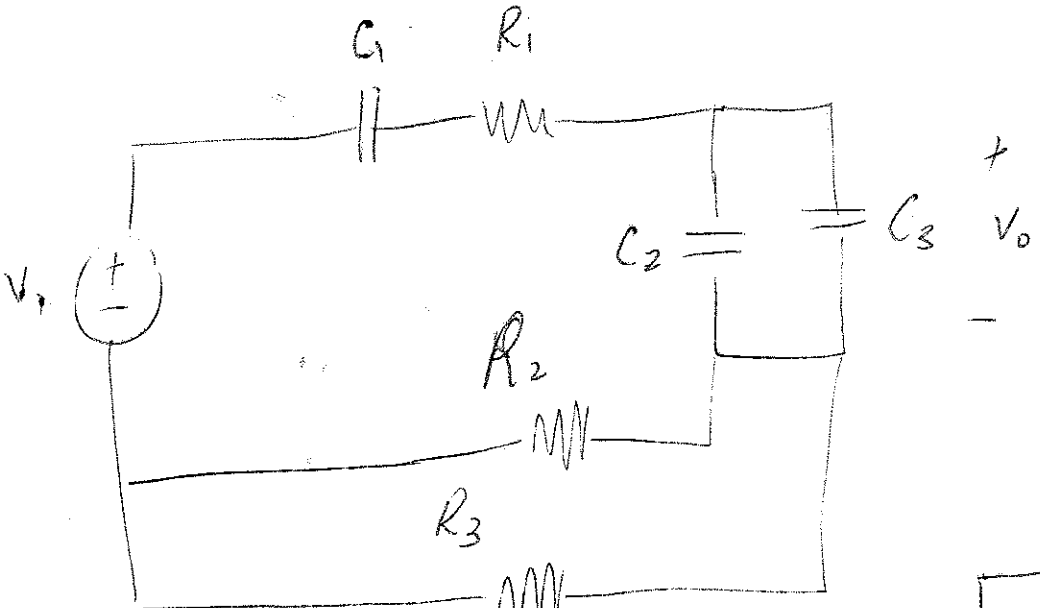
$$V_o(t) = 5 - 3e^{-t/\tau}$$

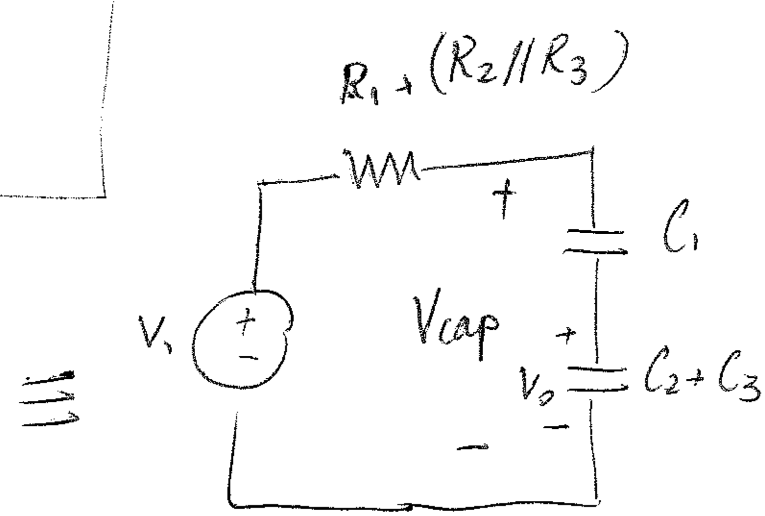
but now τ is 10x greater.



$\tau_2 = 10\,\tau_1$
since $R_2 = 10R_1$

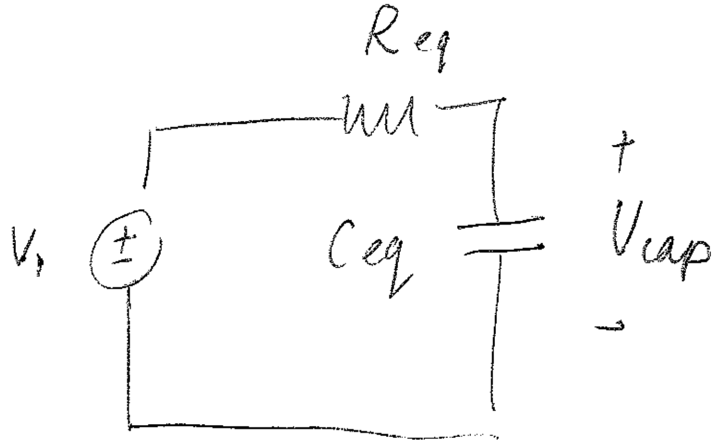
Slide 9:





$$V_0 = \frac{C_{eq}}{C_2 + C_3} V_{cap}$$

So we can further simplify the CKT:

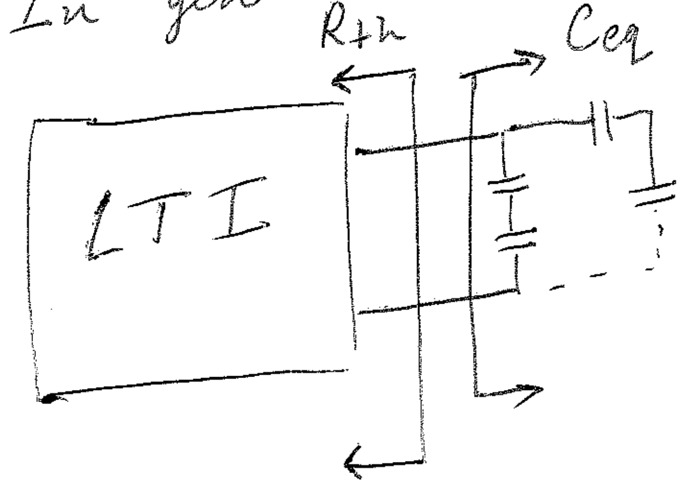


$V_1 = u(t)$

for $t \geq 0$: $V_{cap}(t) = 1 - 1e^{-t/\tau}$ where $\tau = R_{eq} \cdot C_{eq}$

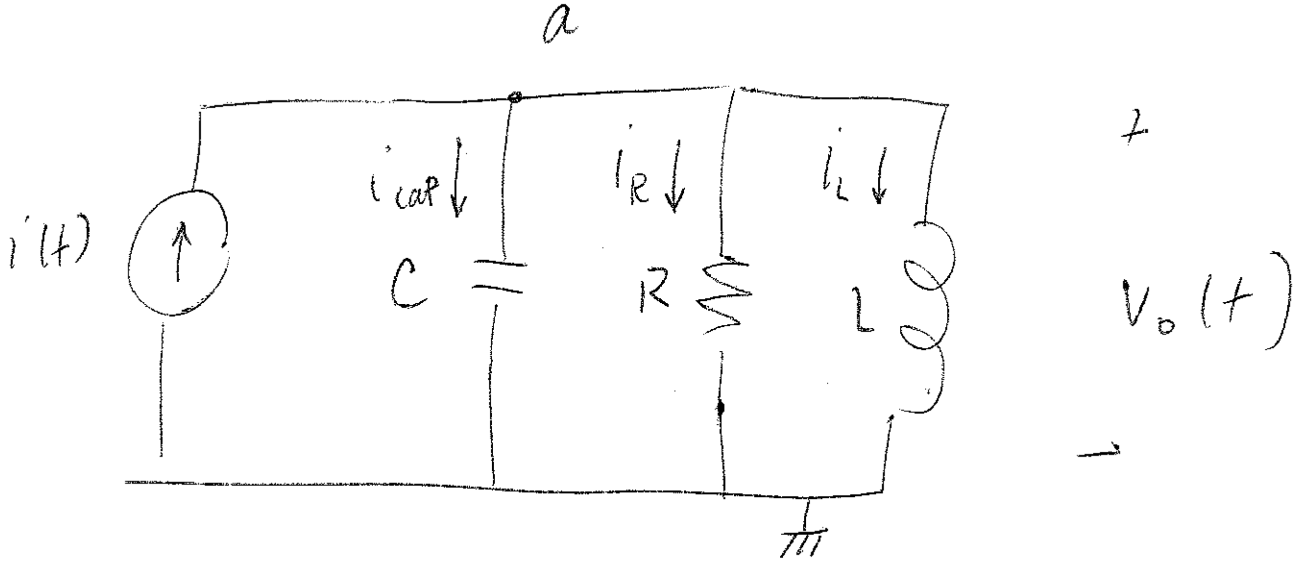
$$\Rightarrow V_0(t) = \frac{C_{eq}}{C_2 + C_3}\left[1 - 1e^{-t/\tau}\right] \text{ for } t \geq 0$$

In general:



$$\underline{\tau = R_{th} \cdot C_{eq}}$$

Slide 10



KCL @ a:

$$-i(t) + i_{cap} + i_R + i_L = 0$$

$$C\frac{dV_o(t)}{dt} + \frac{V_o(t)}{R} + \frac{1}{L}\int V_o(t)\,dt + i_L(0) = i(t)$$

$$\frac{d^2 V_o(t)}{dt^2} + \frac{dV_o(t)}{dt}\cdot\frac{1}{RC} + \frac{1}{LC}V_o(t) = \frac{1}{L}\frac{di(t)}{dt}$$

characteristic equation:

$$\frac{d^2 V_o(t)}{dt^2} + \frac{dV_o(t)}{dt}\frac{1}{RC} + \frac{1}{LC} = 0$$

$$\equiv \frac{d^2 V_o(t)}{dt^2} + \frac{dV_o(t)}{dt}\cdot 2\zeta\omega_0 + \omega_0^2 = 0$$

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\zeta = \frac{1}{2RC\,\omega_0} = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

<u>Slide 11</u>: $\quad i(t) = 1 + u(t) + A\cos(\omega_0 t + \phi)$

split into 2 parts:

$$i_{sin}(t) = A\cos(\omega_0 t + \phi)$$

$$i_{transient}(t) = 1 + u(t)$$

and solve each one seperately. Add in the end by superposition.

a) $i(t) = 1 + u(t)$

$L = 10\mu H, \; C = 1nF, \; R = 50\Omega$

$\zeta = 1 \Rightarrow$ the solution is of the form:

$v_o(t) = K_1 + K_2 e^{-st} + K_3 t e^{-st}$ where $s = \omega_o$

Now use the initial conditions to solve for K_1, K_2 and K_3:

$V_o(0) = 0$ because of the cap:

$K_1 + K_2 = 0$

$V_o(\infty) = 0$ because of the inductor

$\Rightarrow K_1 = 0 \quad \Rightarrow K_2 = 0$

$i_{cap}(0) = 1$ since the step current $u(t)$ will all initially flow in the cap. The DC 1 A always flows through the inductor and does not cause a transient.

$i_{cap} = C \dfrac{dV_o(t)}{dt}$

$= C \dfrac{d\,K_3 t e^{-st}}{dt}$

$$i_{cap}(t) = K_3 C \left[e^{-st} + (-s) t e^{-st} \right]$$

$$i_{cap}(0) = K_3 C e^{-s(0)} = K_3 C = 1$$

$$\Rightarrow K_3 = \frac{1}{C}$$

$$\Rightarrow V_0 = \frac{1}{C} t e^{-\omega_0 t}$$