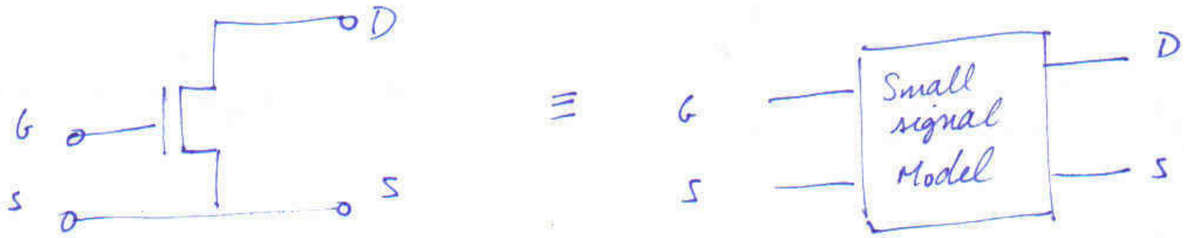
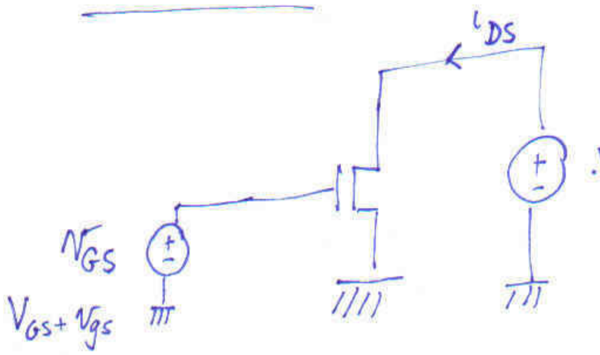
Model of Transistor : Lecture 13



1st Setup :



bias point

desired signal: sine wave

$$i_{DS} = I_{DS} + i_{ds}$$

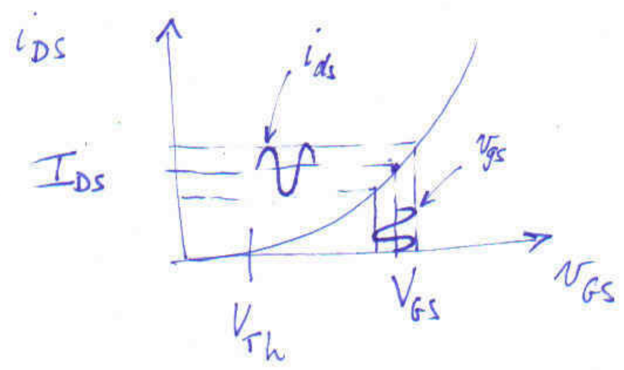
we know:

$$I_{DS} = \mu_n C_{ox} \frac{W}{2L} \left(V_{GS} - V_{Th}\right)^2$$

(assuming $V_o > V_{GS} - V_{Th}$)

now how do we find i_{ds}?

we know that i_{ds} is a function of v_{gs} :



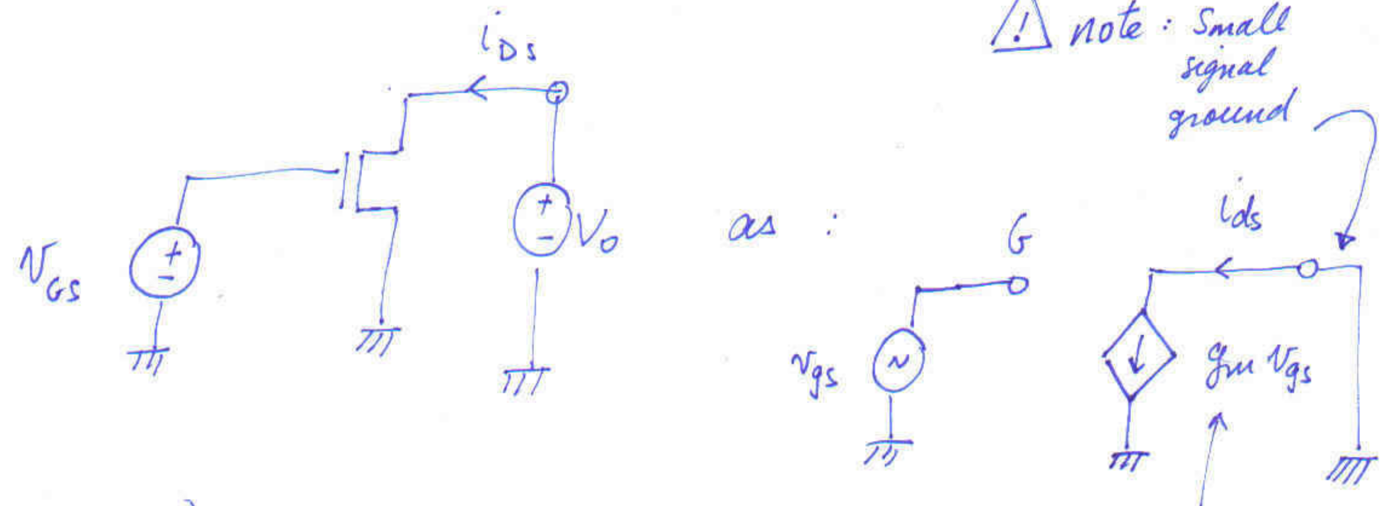
We can linearize the i_{DS} vs. v_{GS} curve around I_{DS}, V_{GS} and find the slope :

$$g_m = \frac{\partial i_{DS}}{\partial v_{GS}}$$

. for v_{gs} small enough :

$$i_{ds} \simeq g_m \cdot v_{gs}$$

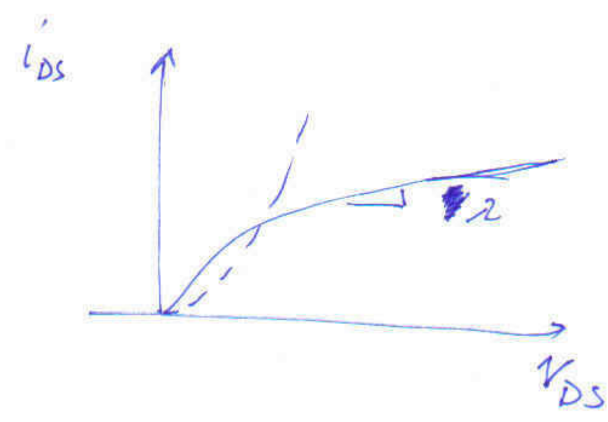
So we can therefore model:



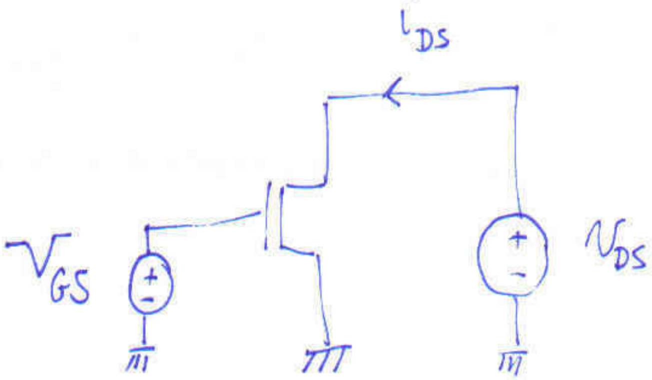
$$g_m = \frac{\partial i_{DS}}{\partial v_{GS}} \cong \frac{W}{L} \mu_n C_{ox} \underbrace{(V_{GS} - V_T)}$$

the bias information is reflected in g_m.

2nd Setup: In the 1st experiment, we kept V_{DS} constant. We know that i_{DS} changes with V_{DS} as well:

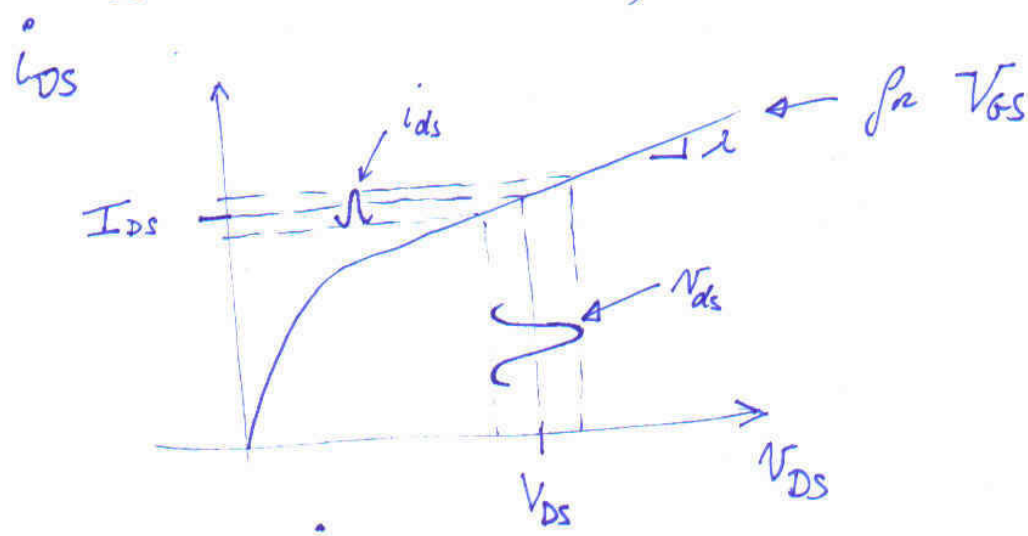


2^{nd} Setup:



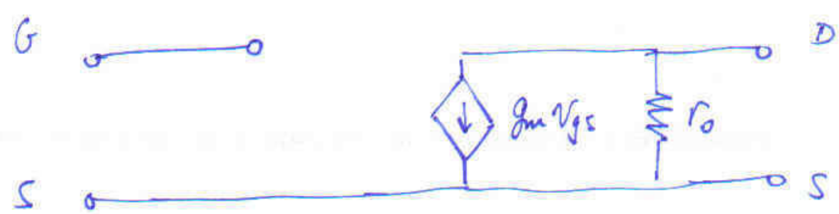
Assume V_{DS} is large enough to put transistor in saturation: $V_{DS} > V_{GS} - V_{Th}$

$$v_{DS} = V_{DS} + v_{ds} \quad , \quad i_{DS} = I_{DS} + i_{ds}$$



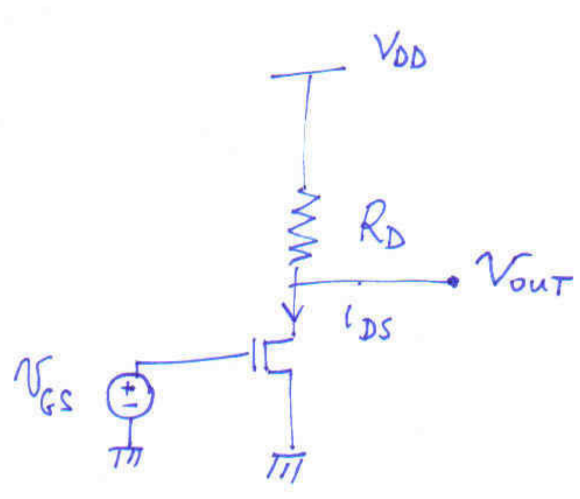
$$g_o = \frac{\partial i_{DS}}{\partial v_{DS}} \cong \lambda I_{DS}$$

$\Rightarrow i_{ds} = g_o \cdot v_{ds}$ or $r_o = \frac{1}{g_o}$ and we can write our complete model:



⚠ only for small signals!

Application :



$$v_{OUT} = V_{DD} - R_D \cdot i_{DS}$$

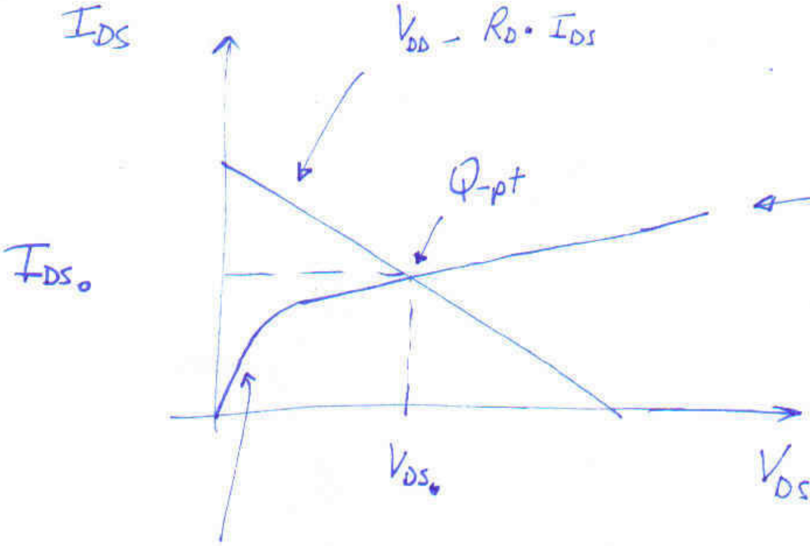
$$= \underbrace{V_{DD} - R_D \cdot I_{DS}}_{\text{DC Bias}} - \underbrace{R_D\, i_{ds}}_{\text{Small signal}}$$

$$v_{OUT} = V_{OUT} + v_{out}$$

$$\Rightarrow V_{OUT} = V_{DD} - R_D \cdot I_{DS}$$

how do we find the DC operating Pt or Q-pt.?

$\rightarrow$ load line: $\underbrace{V_{OUT} = V_{DS} = V_{DD} - R_D \cdot I_{DS}}_{\text{load line equation}}$



$\leftarrow$ for our V_{GS}.

⚠ note we are only looking @ DC values now. We will add the small signal by superposition

So now we have found our $\underline{Q\text{-pt}}$.
next, we can find our small signal model:

$$g_m = \frac{\partial i_{DS}}{\partial v_{GS}} = \frac{W}{L}\mu_n C_{ox}\left(V_{GS}-V_{Th}\right)$$

$$r_o = \left(\frac{\partial i_{DS}}{\partial v_{DS}}\right)^{-1} = \frac{1}{\lambda I_{DS}}$$

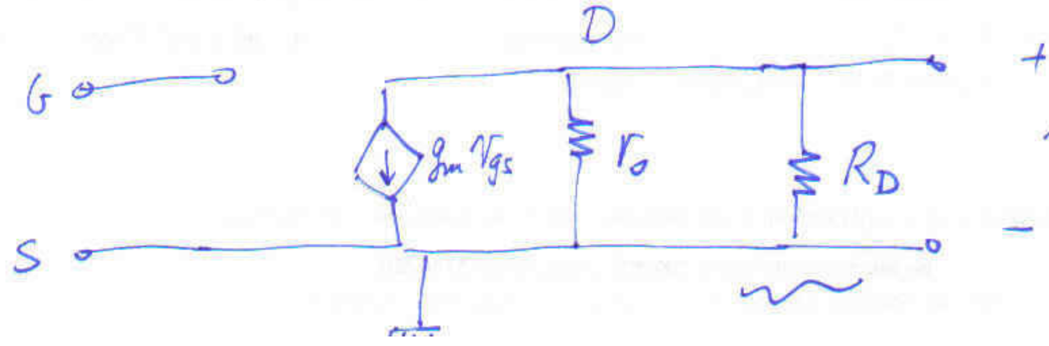
So now we have $v_{out} = -R_D \cdot i_{ds}$

$$= -R_D \cdot \left[g_m v_{gs} + \frac{v_{out}}{r_o}\right]$$

$$v_{out}\left[1 + \frac{+R_D}{r_o}\right] = -R_D \cdot g_m v_{gs}$$

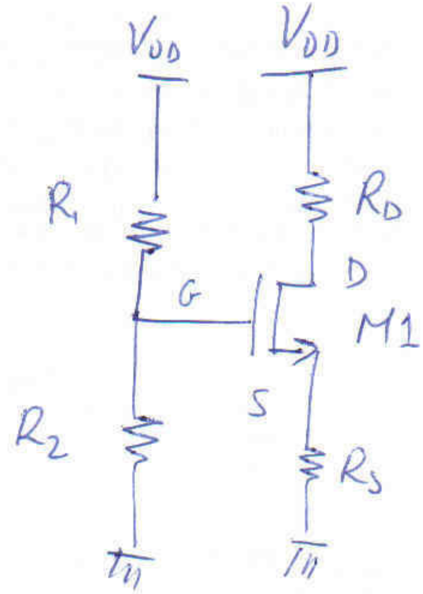
$$v_{out} = \underbrace{\frac{-r_o R_D}{r_o + R_D}}_{r_o // R_D} \cdot g_m v_{gs}$$

more easily we could just have drawn



Lecture 14, slide 4:

1) DC Bias pt:

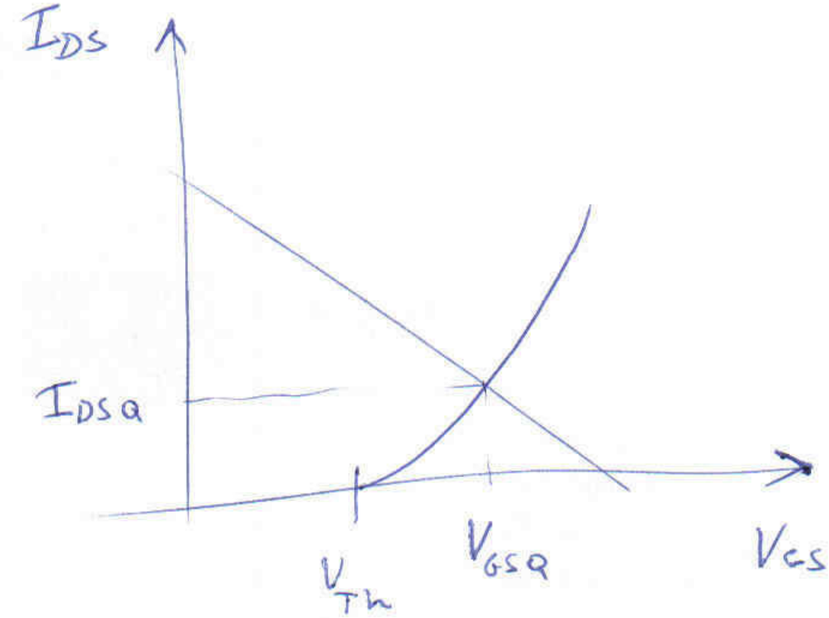


$$V_G = \frac{R_2}{R_1 + R_2} V_{DD}$$

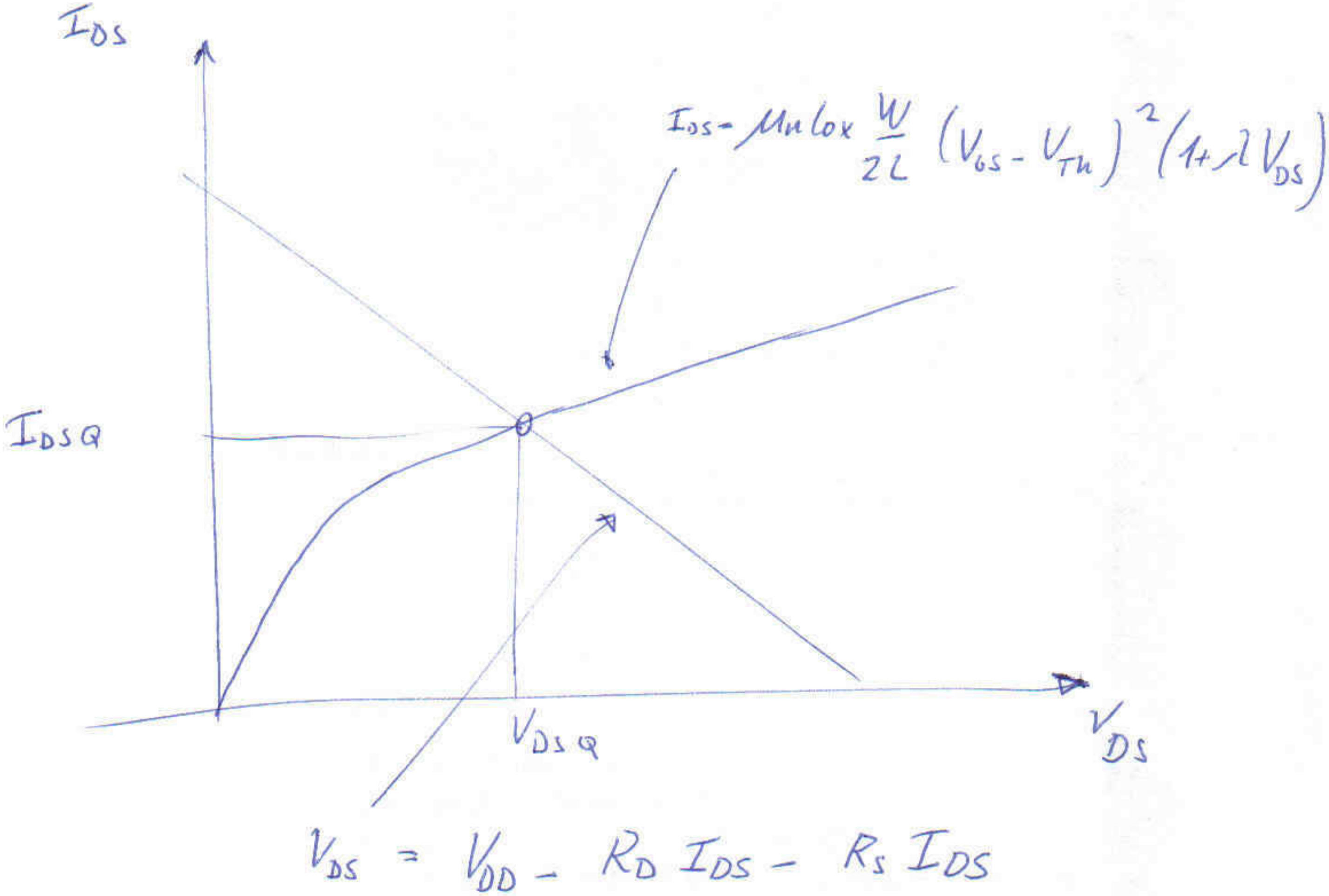
$$V_S = R_S \cdot I_{DS}$$

→ assume M1 is in saturation region

$$\Rightarrow I_{DS} = \mu_n C_{ox} \frac{W}{2L} (V_{GS} - V_{th})^2$$



Find V_{DSQ}:



Note, this value of I_{DSQ} will be slightly different from the one found in the previous graph. Do not worry about this small error since it won't affect the value of g_m greatly.

2) Double check M1 is in saturation : $V_{DSQ} > V_{GS} - V_{Th}$.

if $\lambda = 0$,

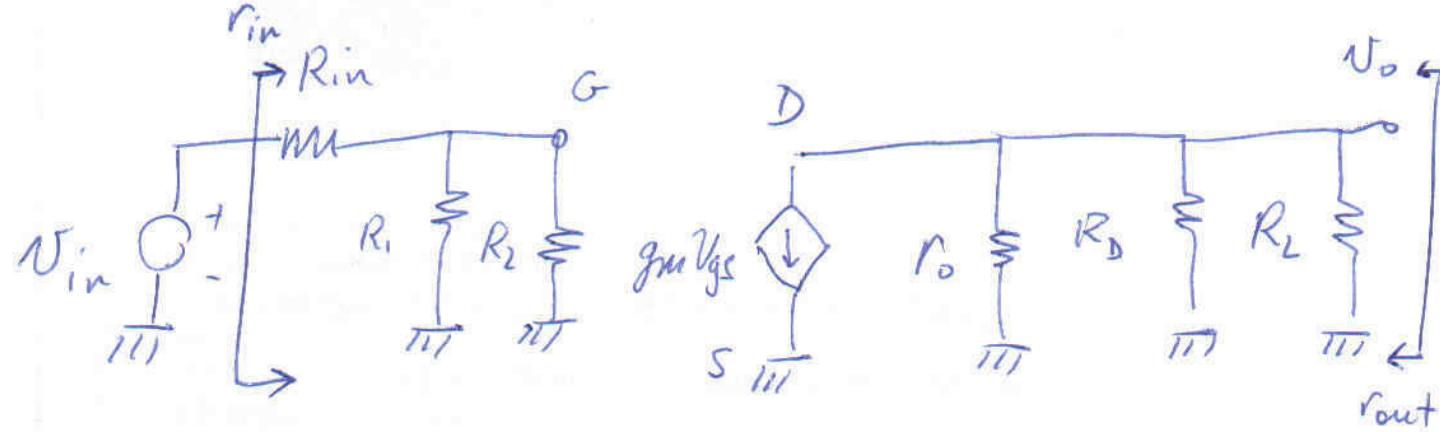
$$V_{DSQ} = V_{DD} - R_D \cdot I_{DSQ} - R_S \cdot I_{DSQ} > V_{GS} - V_{Th}$$

3) Find small signal parameters :

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GSQ} - V_{Th}) \leftarrow \text{in saturation}$$

$$r_o = \frac{1}{\lambda I_{DSQ}}$$

) Small signal model : Capacitors are short :



$$\frac{v_o}{v_{in}} = \frac{R_1 \| R_2}{R_{in} + (R_1 \| R_2)} \cdot (-g_m) \cdot (r_o \| R_D \| R_L)$$

$r_{in} = R_{in} + (R_1 \| R_2)$ $\quad r_{out} = r_o \| R_D \| R_L$

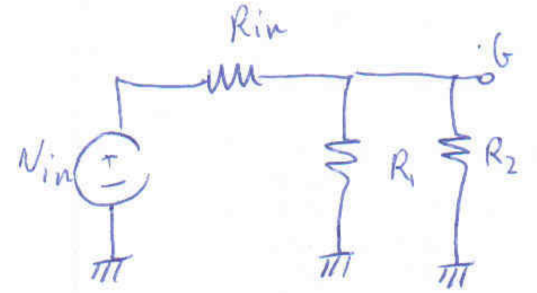
Slide 9

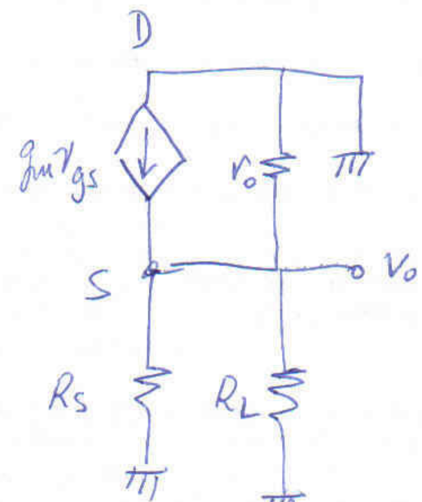
1) DC Bias pt: same as before

2) Double check region of operation of M1:
→ same as before

3) Small signal parameters: same as before

4) Small signal model:



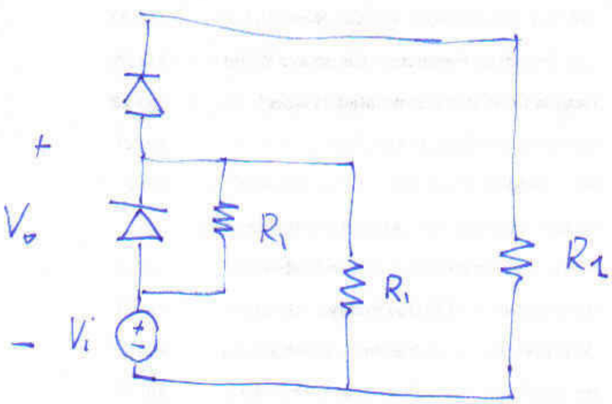


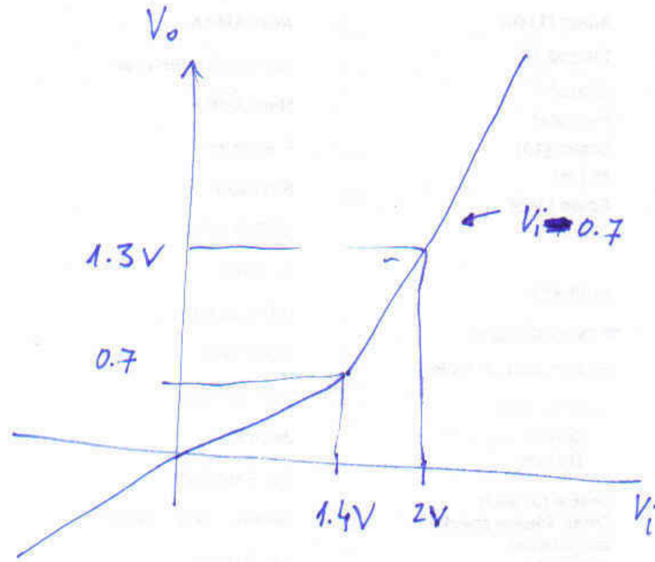
$$\frac{v_g}{v_{in}} = \frac{R_1 \parallel R_2}{(R_1 \parallel R_2) + R_{in}}$$

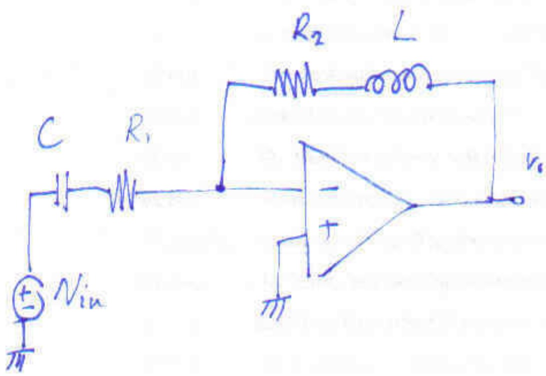
$$V_o = g_m V_{gs} \left(r_o \parallel R_S \parallel R_L \right)$$
$$= g_m V_g \left(r_o \parallel R_S \parallel R_L \right) - g_m V_o \left(R_L \parallel R_S \parallel r_o \right)$$

$$\frac{v_o}{v_g} = \frac{g_m \left(r_o \parallel R_S \parallel R_L \right)}{1 + g_m \left(r_o \parallel R_S \parallel R_L \right)} \simeq 1$$ ← voltage buffer from v_g to v_o

Exam problems:





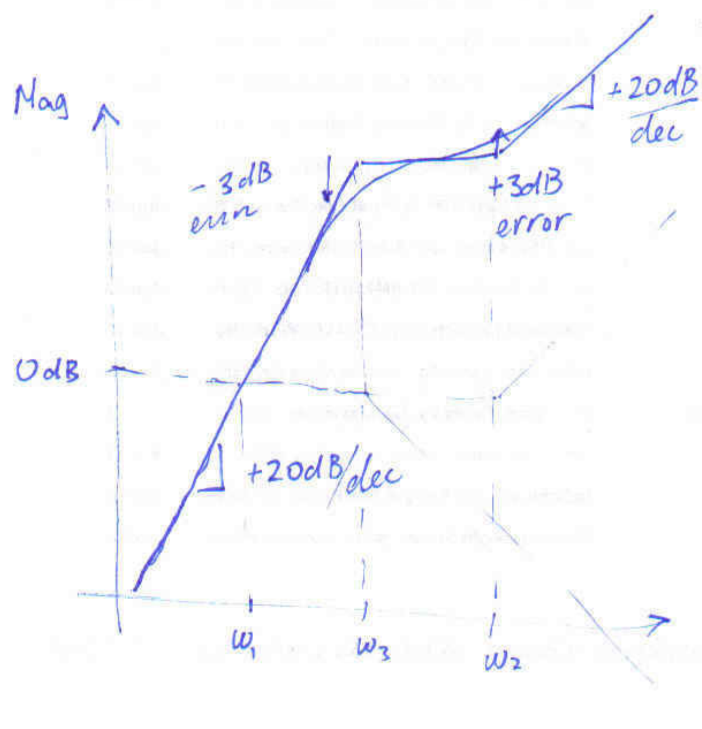


$$\frac{V_o}{V_{in}} = \frac{-(R_2 + j\omega L)}{R_1 + \frac{1}{j\omega C}}$$

$$= \frac{-j\omega C\,(R_2 + j\omega L)}{1 + j\omega R_1 C}$$

$$= \frac{-j\omega R_2 C\,(1 + j\omega L/R_2)}{1 + j\omega R_1 C}$$

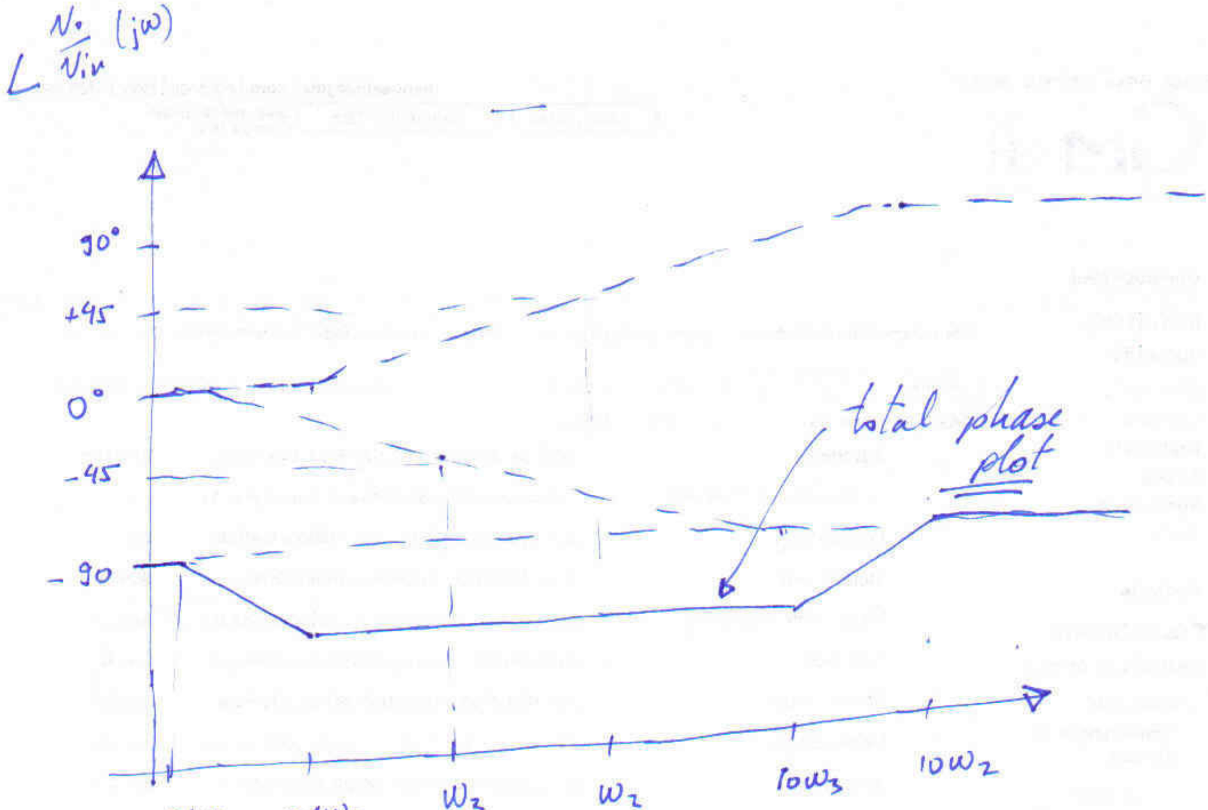
$$= \frac{-j\frac{\omega}{\omega_1}\,(1 + j\omega/\omega_2)}{1 + j\omega/\omega_3}$$

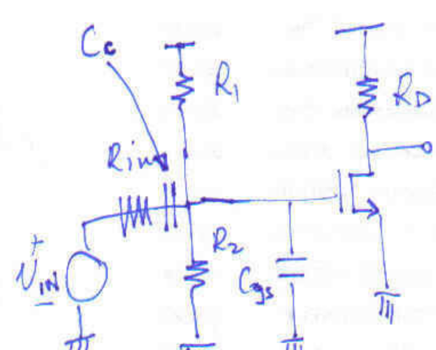


$$\omega_1 = \frac{1}{R_2 C}$$

$$\omega_2 = \frac{R_2}{L}$$

$$\omega_3 = \frac{1}{R_1 C}$$





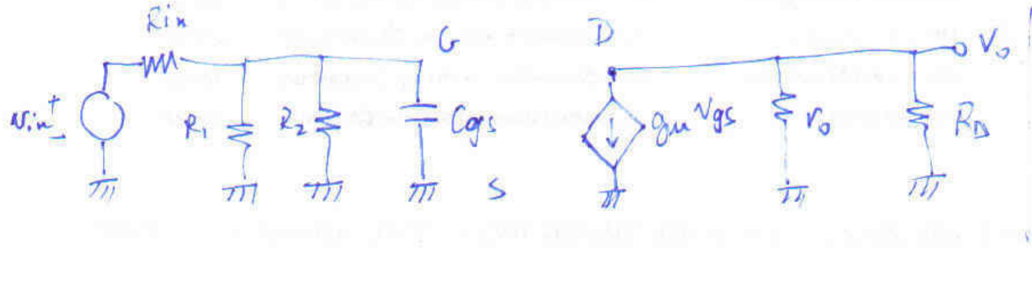
C_c is infinitely large,
C_{gs} is not $\Rightarrow$ include C_{gs} in SSM.

1.) Bias Pt: $V_{GSQ} = V_{DD} \cdot \frac{R_2}{R_1 + R_2}$

2) $g_m = \mu_n C_{ox} \frac{W}{L} (V_{GSQ} - V_{Th})$

$r_o = \frac{1}{\lambda I_{DSQ}}$ and $I_{DSQ} = \mu_n C_{ox} \frac{W}{2L} (V_{GSQ} - V_{Th})^2$

3) Small signal Model:



$$\frac{v_o}{v_{in}} = \frac{R_1 \| R_2 \| \frac{1}{j\omega C_{gs}}}{R_{in} + \left(R_1 \| R_2 \| \frac{1}{j\omega C_{gs}}\right)} \times -(g_m)(R_D \| r_o)$$