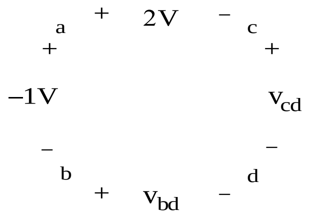
Announcements

- Sections begin this week
  - ☐ Cancelled Sections: Th 12-2.
- Labs begin this week. Attend your only second lab slot this week.
  - ☐ Cancelled labs: ThF 2-5. Please check your Lab section.
- Homework #1 online
  - ☐ Due next Monday at 6pm

Review

Find v_{ab}, v_{ca}, v_{cb}



Note that the labeling convention has nothing to do with whether or not v is positive or negative.

Lecture #2

OUTLINE

- Circuit Elements
- Circuit element *I-V* characteristics
- Construction of a circuit model
- Kirchhoff's laws – a closer look

Reading

(Chapter 1, begin Ch. 2)

Circuit Elements

- 5 ideal basic circuit elements:
  - ☐ voltage source
  - ☐ current source

  } ***active elements***, capable of generating electric energy

  - ☐ resistor
  - ☐ inductor
  - ☐ capacitor

  } ***passive elements***, incapable of generating electric energy

- Many practical systems can be modeled with just sources and resistors
- The basic analytical techniques for solving circuits with inductors and capacitors are similar to those for resistive circuits

Electrical Sources

- An ***electrical source*** is a device that is capable of converting non-electric energy to electric energy and *vice versa*.

  Examples:
  - ☐ battery: chemical ⟷ electric
  - ☐ dynamo (generator/motor): mechanical ⟷ electric

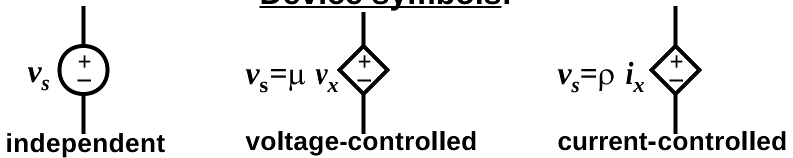
    (Ex. gasoline-powered generator, Bonneville dam)

→ Electrical sources can either deliver or absorb power

Ideal Voltage Source

- Circuit element that maintains a prescribed voltage across its terminals, **regardless of the current flowing in those terminals**.
  - ☐ Voltage is known, but current is determined by the circuit to which the source is connected.
- The voltage can be either **independent** or **dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

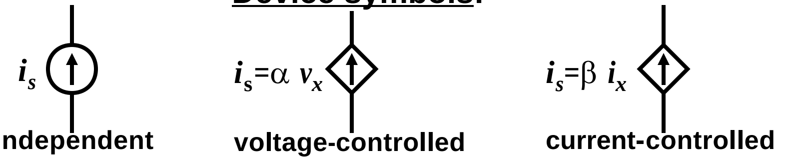
Device symbols:



Ideal Current Source

- Circuit element that maintains a prescribed current through its terminals, **regardless of the voltage across those terminals**.
  - Current is known, but voltage is determined by the circuit to which the source is connected.
- The current can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

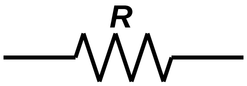
Device symbols:



Electrical Resistance

- ***Resistance:*** the ratio of voltage drop and current. The circuit element used to model this behavior is the ***resistor***.

Circuit symbol:



Units: Volts per Ampere   ohms (Ω)

Georg Simon Ohm
1789-1854

- The current flowing in the resistor is proportional to the voltage across the resistor:

$$v = i\,R$$ (Ohm's Law)

where v = voltage (V), i = current (A), and R = resistance (Ω)

Electrical Conductance

- **Conductance** is the reciprocal of resistance.

Symbol: G

Units: **siemens (S)**

Example:

Consider an 8 Ω resistor. *What is its conductance?*

Werner von Siemens
1816-1892

Short Circuit and Open Circuit

Wire (“short circuit”):

- $R = 0$ → **no voltage difference exists**
  (all points on the wire are at the same potential)
- Current can flow, as determined by the circuit

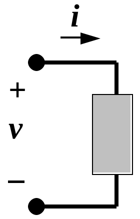
Air (“open circuit”):

- $R = \infty$ → **no current flows**
- Voltage difference can exist, as determined by the circuit

Figure 2.10 Circuit symbols. (a) Short circuit. (b) Open circuit. (c) Switch.

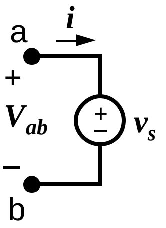
Current *vs.* Voltage (*I-V*) Characteristic

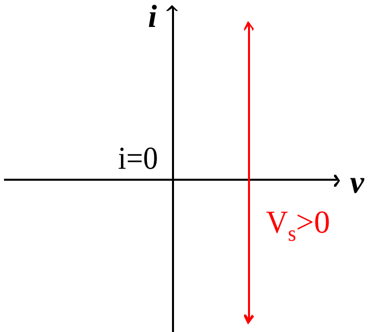
- Voltage sources, current sources, and resistors can be described by plotting the current (i) as a function of the voltage (v)



Passive? Active?

I-V Characteristic of an Ideal Voltage Source

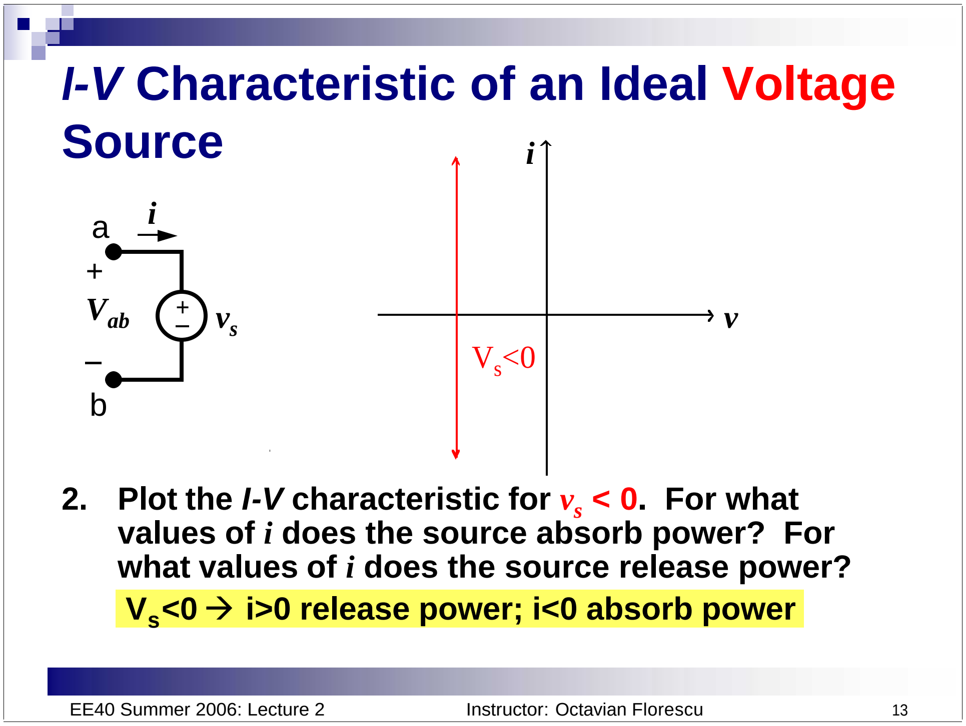




1. **Plot the *I-V* characteristic for $v_s > 0$. For what values of i does the source absorb power? For what values of i does the source release power?**

$V_s>0$ → i<0 release power; i>0 absorb power

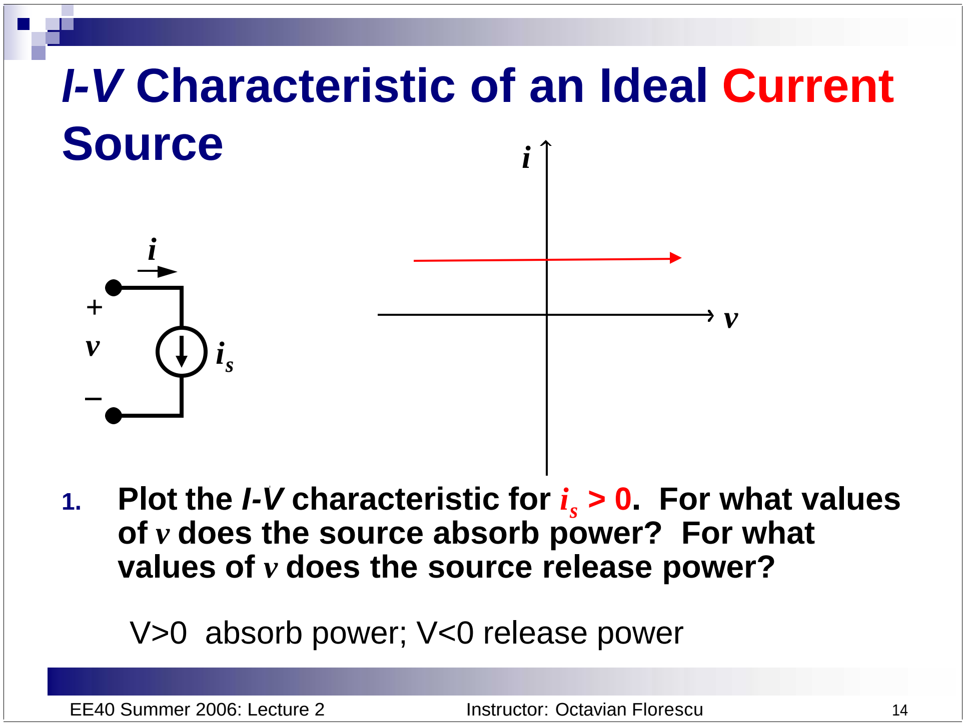
What is the *I-V* characteristic for an ideal wire?



I-V Characteristic of an Ideal Voltage Source

2. Plot the *I-V* characteristic for $v_s < 0$. For what values of i does the source absorb power? For what values of i does the source release power?

   $V_s<0$ → $i>0$ release power; $i<0$ absorb power

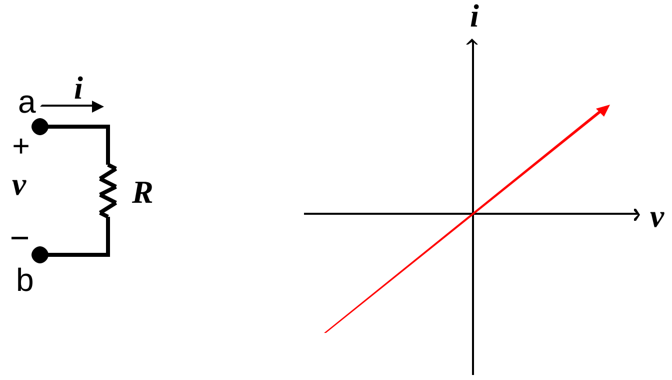


I-V Characteristic of an Ideal Current Source

1. Plot the *I-V* characteristic for $i_s > 0$. For what values of v does the source absorb power? For what values of v does the source release power?

   V>0 absorb power; V<0 release power

I-V Characteristic of Ideal Resistor



1. **Plot the *I-V* characteristic for R = 1 kΩ. What is the slope?**

"Lumped Element" Circuit Modeling

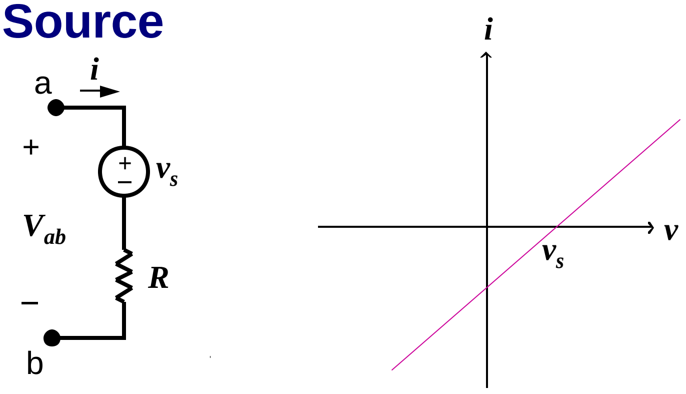
(Model = representation of a real system which simplifies analysis)

- In circuit analysis, important characteristics are grouped together in "lumps" (separate circuit elements) connected by perfect conductors ("**wires**")
- An electrical system can be modeled by an **electric circuit** (combination of paths, each containing 1 or more **circuit elements**).

Construction of a Circuit Model

- The electrical behavior of each physical component is of primary interest.
- We need to account for undesired as well as desired electrical effects.
- Simplifying assumptions should be made wherever reasonable.

I-V Characteristic of a real Voltage Source

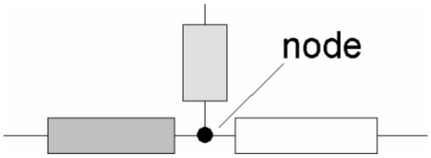


1. What is the *I-V* characteristic for an real current source?

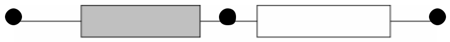
2. What is the *I-V* characteristic for an ideal wire?

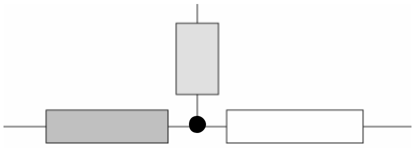
Terminology: Nodes and Branches

Node**:** A point where two or more circuit elements are connected



Branch**:** A path that connects two nodes

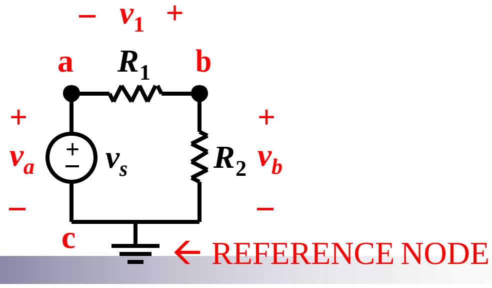

A single branch


NOT a single branch

Notation: Node and Branch Voltages

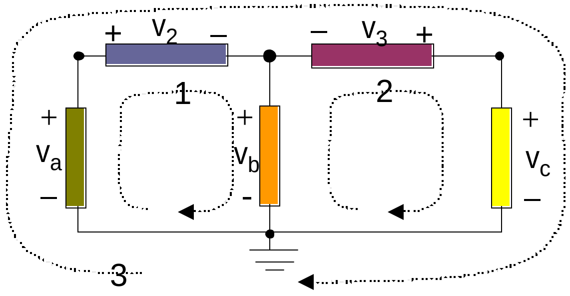
- Use one node as the reference (the “common” or “ground” node) – label it with a symbol
- The voltage drop from node **x** to the reference node is called the **node voltage v$_x$**.
- The voltage across a circuit element is defined as the difference between the node voltages at its terminals

Example:



Circuit Loops

- A ***loop*** is formed by tracing a closed path in a circuit through selected basic circuit elements without passing through any intermediate node more than once. Below are marked loops 1,2 and 3:



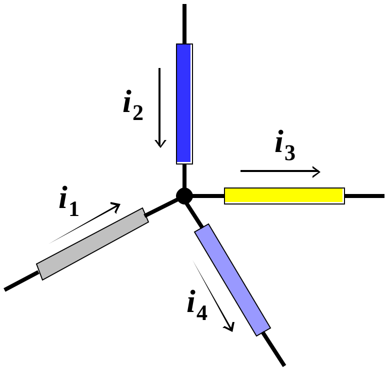
Kirchhoff's Laws

- **Kirchhoff's Current Law (KCL):**
  - The algebraic sum of all the **currents** entering any **node** in a circuit equals zero.
- **Kirchhoff's Voltage Law (KVL):**
  - The algebraic sum of all the **voltages** around any **loop** in a circuit equals zero.

Gustav Robert Kirchhoff
1824-1887

Using Kirchhoff’s Current Law (KCL)

Consider a node connecting several branches:



- Use **reference directions** to determine whether currents are “entering” or “leaving” the node – with no concern about actual current directions

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Formulations of Kirchhoff’s Current Law

(Charge stored **in node** is zero.)

Formulation 1:

Sum of currents entering node
= sum of currents leaving node

Formulation 2:

Algebraic sum of currents entering node = 0

- Currents leaving are included with a minus sign.

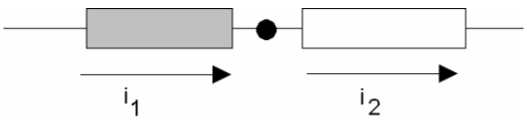
Formulation 3:

Algebraic sum of currents leaving node = 0

- Currents entering are included with a minus sign.

A Major Implication of KCL

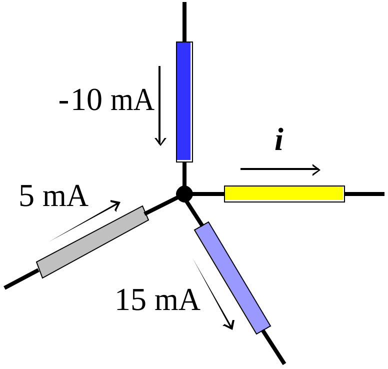
- KCL tells us that **all of the elements in a single branch carry the same current**.
- We say these elements are connected ***in series***.



Current entering node = Current leaving node

$$i_1 = i_2$$

KCL Example



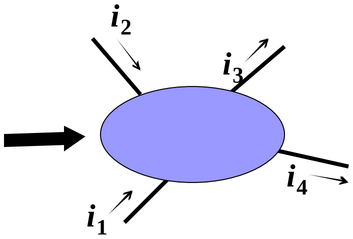
Currents entering the node:

Currents leaving the node:

<u>KCL</u>:

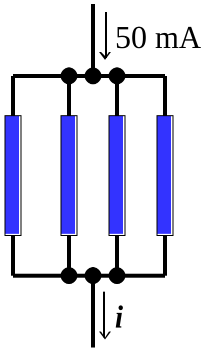
Generalization of KCL

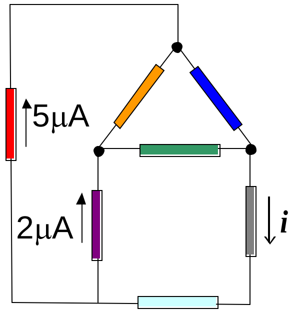
- The sum of currents entering/leaving a **closed surface** is zero. Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

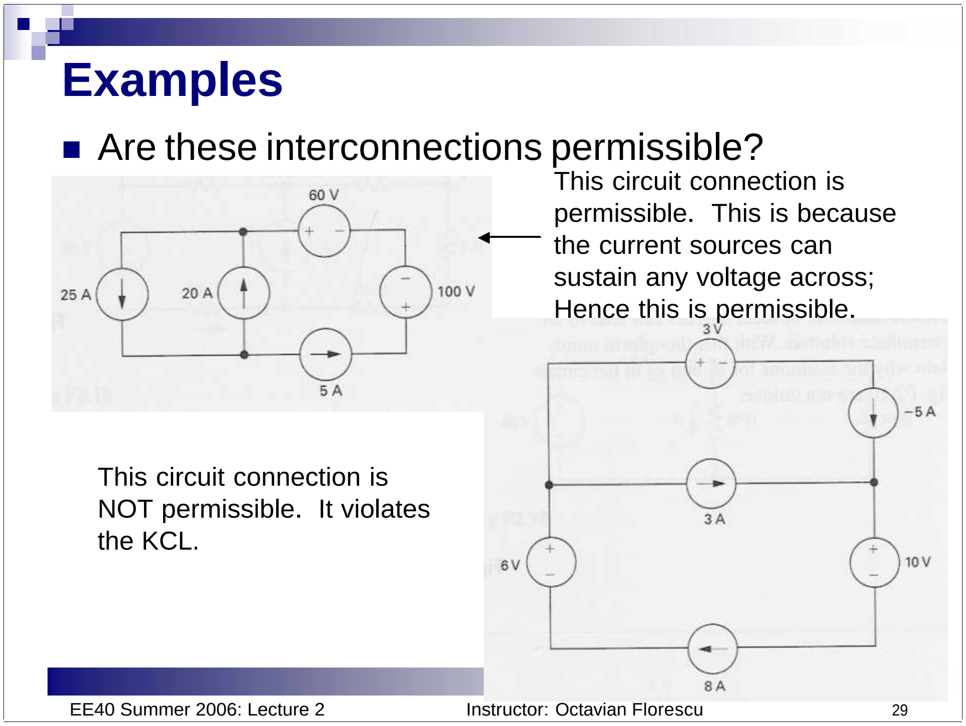


This could be a big chunk of a circuit, *e.g.* a “black box”

Generalized KCL Examples





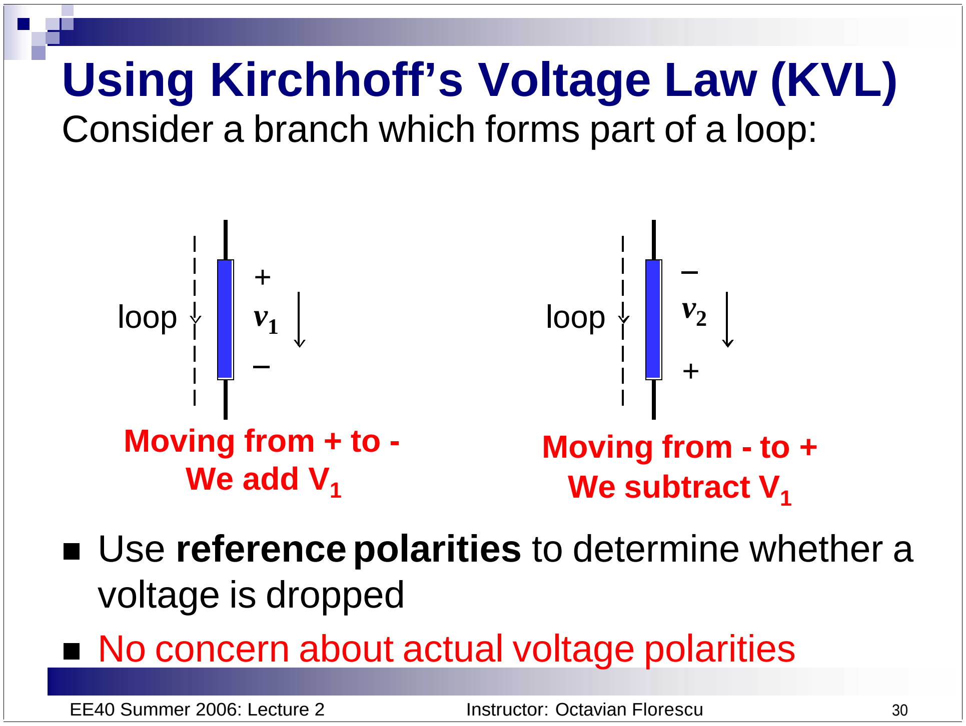


Examples

- Are these interconnections permissible?

This circuit connection is permissible. This is because the current sources can sustain any voltage across; Hence this is permissible.

This circuit connection is NOT permissible. It violates the KCL.



Using Kirchhoff's Voltage Law (KVL)

Consider a branch which forms part of a loop:

Moving from + to -
We add V_1

Moving from - to +
We subtract V_1

- Use **reference polarities** to determine whether a voltage is dropped
- No concern about actual voltage polarities

Formulations of Kirchhoff's Voltage Law (Conservation of energy)

Formulation 1:

Sum of voltage drops around loop
= sum of voltage rises around loop

Formulation 2:

Algebraic sum of voltage drops around loop = 0

- Voltage rises are included with a minus sign.

(Handy trick: Look at the first sign you encounter on each element when tracing the loop.)

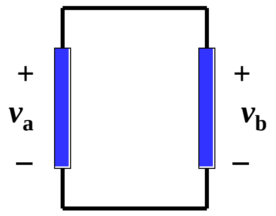
Formulation 3:

Algebraic sum of voltage rises around loop = 0

- Voltage drops are included with a minus sign.

A Major Implication of KVL

- KVL tells us that **any set of elements which are connected at both ends carry the same voltage**.
- We say these elements are connected **in parallel**.

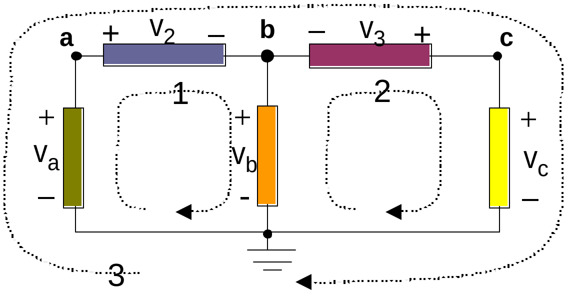


Applying KVL in the clockwise direction, starting at the top:

$$v_b - v_a = 0 \rightarrow v_b = v_a$$

KVL Example

Three closed paths:



<u>Path 1</u>:

<u>Path 2</u>:

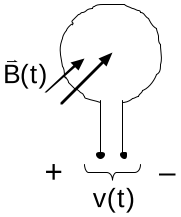
<u>Path 3</u>:

An Underlying Assumption of KVL

- No time-varying magnetic flux through the loop

  Otherwise, there would be an induced voltage (Faraday's Law)

- <u>Note</u>: Antennas are designed to "pick up" electromagnetic waves; "regular circuits" often do so undesirably.



Avoid these loops!

How do we deal with antennas (EECS 117A)?

Include a voltage source as the circuit representation of the induced voltage or "noise".

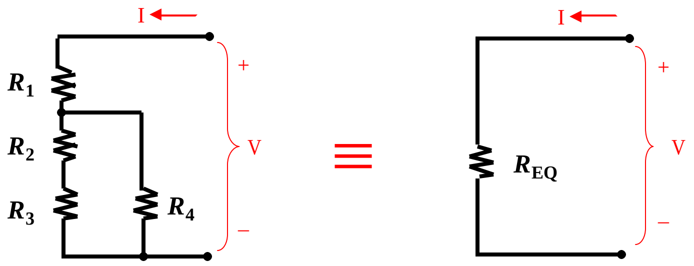
(Use a lumped model rather than a distributed (wave) model.)

Tips for KCL, KVL

- Find one formulation which works for you and stick with it.
- I suggest:
  - KCL
    - Algebraic sum of currents leaving node = 0
    - Currents entering are included with a minus sign.
  - KVL
    - Algebraic sum of voltage drops around loop = 0
    - Voltage rises are included with a minus sign.

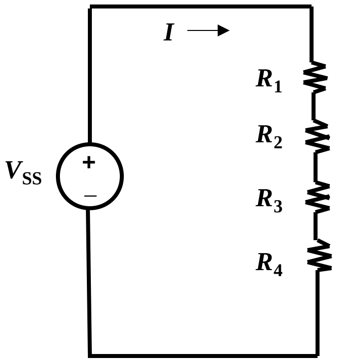
Equivalent Resistance

- Arrangements of several resistors can be lumped into one equivalent resistance:



Resistors in Series

Consider a circuit with multiple resistors connected in series. Find their equivalent resistance.



- KCL tells us that the same current (I) flows through every resistor
- KVL tells us:

Equivalent resistance of series resistors is the sum of resistances

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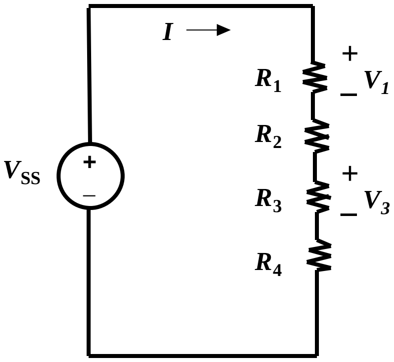
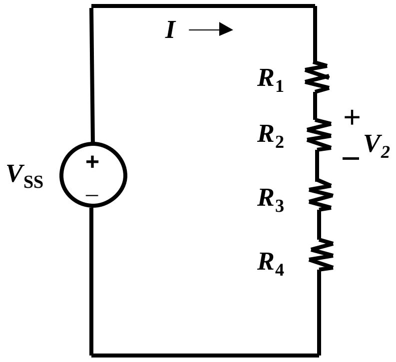
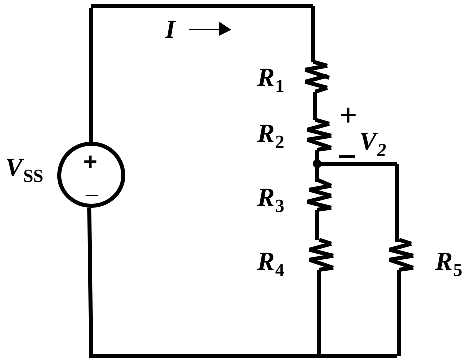
Voltage Divider

$$I = V_{SS} / (R_1 + R_2 + R_3 + R_4) \ldots$$



When can the Voltage Divider Formula be Used?





$$V_2 = \frac{R_2}{R_1+R_2+R_3+R_4} \cdot V_{SS}$$

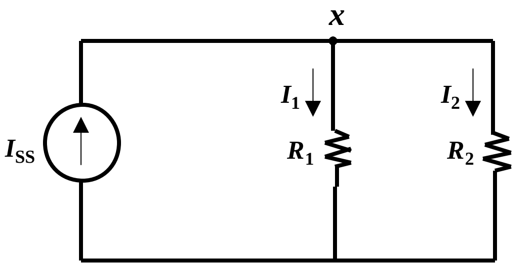
Correct, if nothing else
is connected to nodes

$$V_2 \; \frac{R_2}{R_1+R_2+R_3+R_4} \cdot V_{SS}$$

Why? What is V_2?

Resistors in Parallel

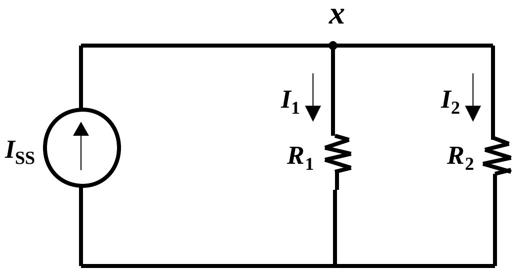
Consider a circuit with two resistors connected in parallel.
Find their equivalent resistance.



- KVL tells us that the same voltage is dropped across each resistor
- KCL tells us:

Equivalent <u>conductance</u> of parallel resistors is the sum of conductances

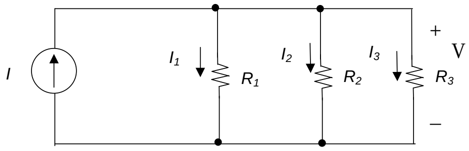
Current Divider



$$V_x = I_1 R_1 = I_2 R_2 \ldots$$

Generalized Current Divider Formula

Consider a current divider circuit with >2 resistors in parallel:



$$V = \frac{I}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right)}$$

$$I_3 = \frac{V}{R_3} = I\left[\frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3}\right]$$

Example: Power Absorbed by a Resistor

$$p = vi = (iR)i = i^2R$$

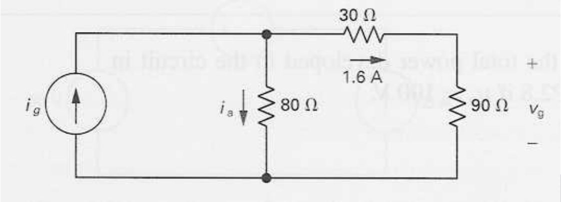
$$p = vi = v(v/R) = v^2/R$$

Note that $p > 0$ always, for a resistor → a resistor dissipates electric energy

Example:

a) Calculate the voltage v_g and current i_a.

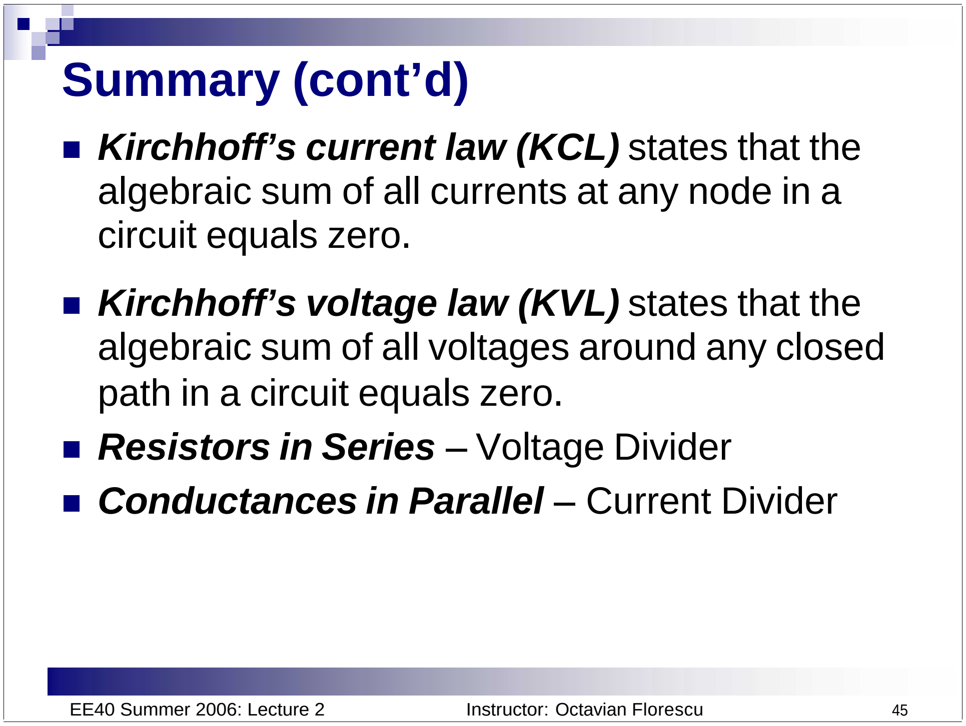
b) Determine the power dissipated in the 80Ω resistor.



Summary

- An ***ideal voltage source*** maintains a prescribed voltage regardless of the current in the device.
- An ***ideal current source*** maintains a prescribed current regardless of the voltage across the device.
- A ***resistor*** constrains its voltage and current to be proportional to each other:

$$v = iR \quad \text{(Ohm's law)}$$



Summary (cont’d)

- ***Kirchhoff’s current law (KCL)*** states that the algebraic sum of all currents at any node in a circuit equals zero.
- ***Kirchhoff’s voltage law (KVL)*** states that the algebraic sum of all voltages around any closed path in a circuit equals zero.
- ***Resistors in Series*** – Voltage Divider
- ***Conductances in Parallel*** – Current Divider