

## Announcements

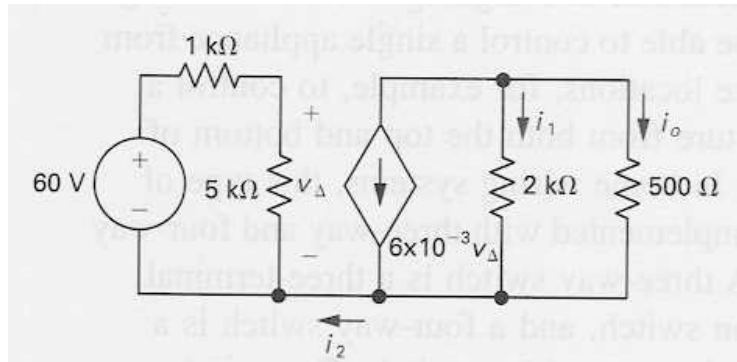
- Homework #1 due Monday at 6pm
  - White drop box in Student Lounge on the second floor of Cory
- Tuesday labs cancelled next week
  - Attend your other lab slot
- Books on reserve in Bechtel
  - Hambley, 2<sup>nd</sup> and 3<sup>rd</sup> Edition

## Review from Last Class

- KCL, KVL
  - Node and Loops
- Resistors in Series and parallel
  - Equivalent Resistance
- Voltage and Current Division
- I-V Characteristics
  - Sources
  - Resistors

## Circuit w/ Dependent Source Example

Find  $i_2$ ,  $i_1$  and  $i_o$



## Lecture #3

### OUTLINE

- KCL, KVL Examples
- Thevenin/Norton Equivalent circuit
- Measurement Devices

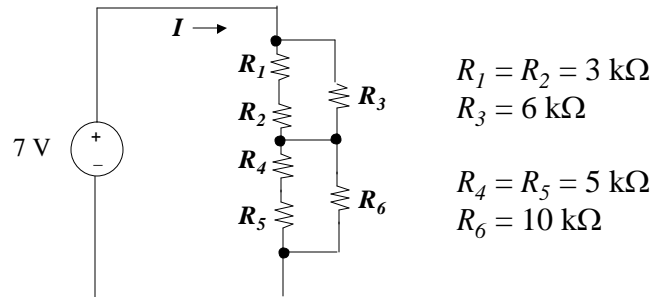
### Reading

Finish Chapter 1,2

## Using Equivalent Resistances

Simplify a circuit before applying KCL and/or KVL:

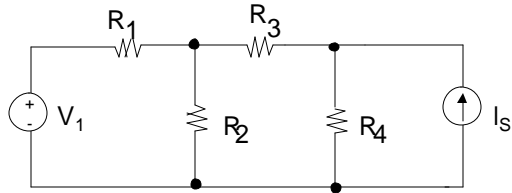
Example: Find  $I$



## Node-Voltage Circuit Analysis Method

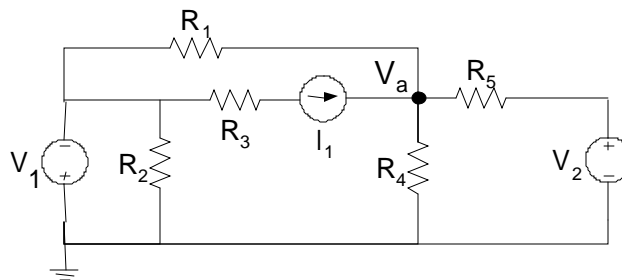
1. **Choose a reference node** ("ground")  
*Look for the one with the most connections!*
2. **Define unknown node voltages**  
*those which are not fixed by voltage sources*
3. **Write KCL at each unknown node**, expressing current in terms of the node voltages (using the  $I$ - $V$  relationships of branch elements)  
*Special cases: floating voltage sources*
4. **Solve the set of independent equations**  
 $N$  equations for  $N$  unknown node voltages

## Nodal Analysis: Example #1



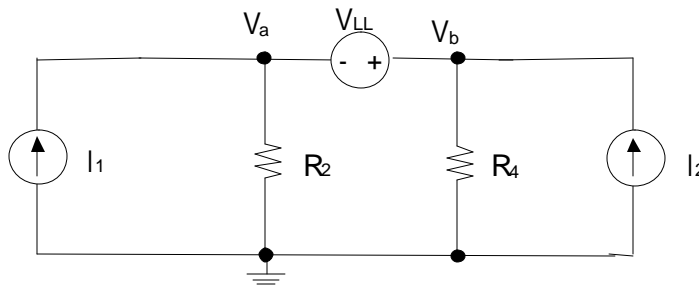
1. Choose a reference node.
2. Define the node voltages (except reference node and the one set by the voltage source).
3. Apply KCL at the nodes with unknown voltage.
4. Solve for unknown node voltages.

## Nodal Analysis: Example #2



## Nodal Analysis w/ “Floating Voltage Source”

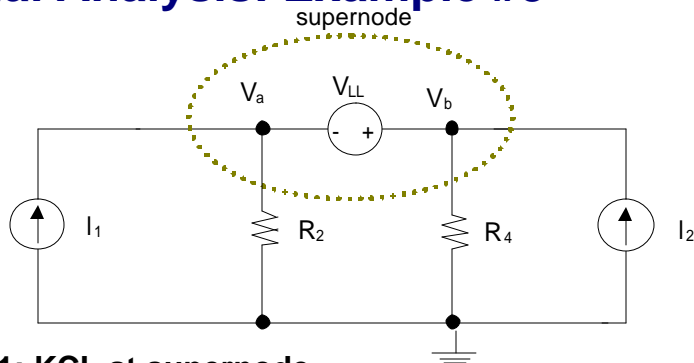
A “floating” voltage source is one for which neither side is connected to the reference node, e.g.  $V_{LL}$  in the circuit below:



## Nodal Analysis w/ “Floating Voltage Source”

- Problem: We cannot write KCL at nodes  $a$  or  $b$  because there is no way to express the current through the voltage source in terms of  $V_a - V_b$ .
- Solution: Define a “supernode” – that chunk of the circuit containing nodes  $a$  and  $b$ . Express KCL for this supernode. Incorporate voltage source constraint into KCL equation.

## Nodal Analysis: Example #3



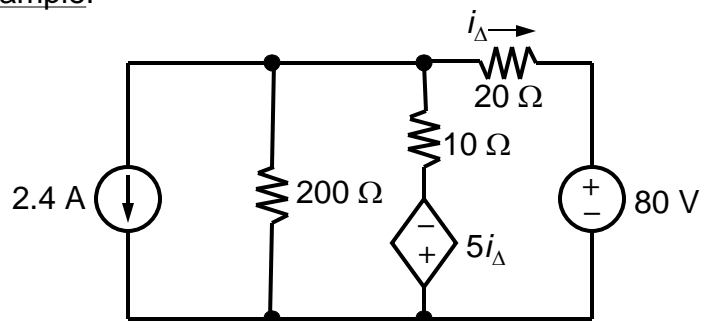
**Eq'n 1: KCL at supernode**

**Substitute property of voltage source:**

## Node-Voltage Method and Dependent Sources

- If a circuit contains dependent sources, what to do?

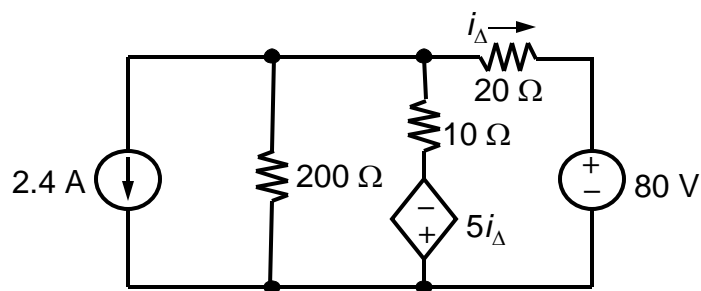
Example:



## Node-Voltage Method and Dependent Sources

- Dependent current source: treat as independent current source in organizing and writing node eqns, but include (substitute) constraining dependency in terms of defined node voltages.
- Dependent voltage source: treat as independent voltage source in organizing and writing node eqns, but include (substitute) constraining dependency in terms of defined node voltages.

### Example:



## Mesh Circuit Analysis Method

- 1) **Select  $M$  independent mesh currents** such that at least one mesh current passes through each branch\*

$$M = \text{\#branches} - \text{\#nodes} + 1$$

- 2) **Apply KVL to each mesh**, expressing voltages in terms of mesh currents

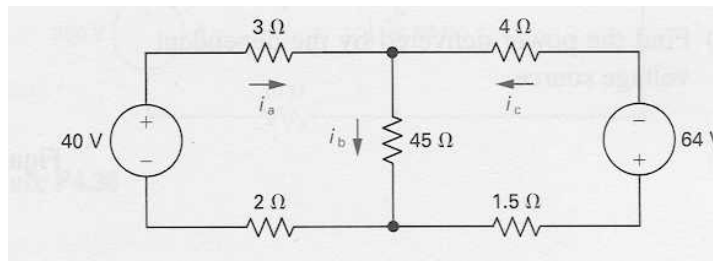
=>  $M$  equations for

$M$  unknown mesh currents

- 3) **Solve for mesh currents**

=> determine node voltages

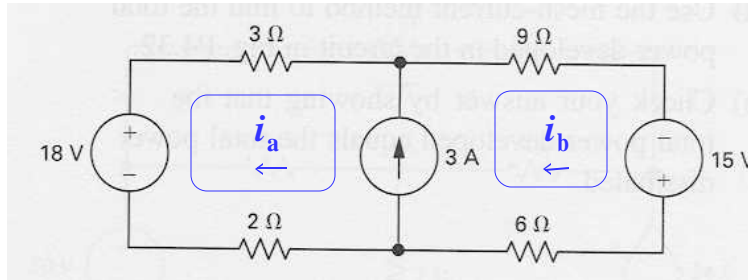
## Mesh Analysis: Example #1



1. Select  $M$  mesh currents.
2. Apply KVL to each mesh.
3. Solve for mesh currents.



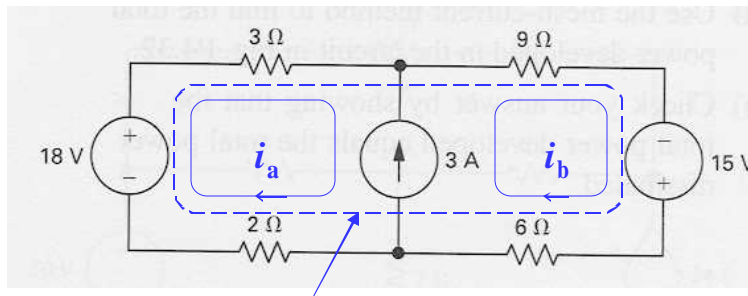
## Mesh Analysis with a Current Source



**Problem:** We cannot write KVL for meshes a and b because there is no way to express the voltage drop across the current source in terms of the mesh currents.

**Solution:** Define a “supermesh” – a mesh which avoids the branch containing the current source. Apply KVL for this supermesh.

## Mesh Analysis: Example #2



**Eq'n 1:** KVL for supermesh

**Eq'n 2:** Constraint due to current source:

## Mesh Analysis with Dependent Sources

- Exactly analogous to Node Analysis
- Dependent Voltage Source: (1) Formulate and write KVL mesh eqns. (2) Include and express dependency constraint in terms of mesh currents
- Dependent Current Source: (1) Use supermesh. (2) Include and express dependency constraint in terms of mesh currents

## Formal Circuit Analysis Methods

### NODAL ANALYSIS

("Node-Voltage Method")

- 1) Choose a reference node
- 2) Define unknown node voltages
- 3) Apply KCL to each unknown node, expressing current in terms of the node voltages
  - => N equations for N unknown node voltages
- 4) Solve for node voltages
  - => determine branch currents

### MESH ANALYSIS

("Mesh-Current Method")

- 1) Select M independent mesh currents such that at least one mesh current passes through each branch\*
  - $M = \# \text{branches} - \# \text{nodes} + 1$
- 2) Apply KVL to each mesh, expressing voltages in terms of mesh currents
  - => M equations for M unknown mesh currents
- 3) Solve for mesh currents
  - => determine node voltages

## Superposition

A **linear circuit** is one constructed only of linear elements (linear resistors, and linear capacitors and inductors, linear dependent sources) and independent sources. Linear means I-V characteristic of elements/sources are straight lines when plotted

### Principle of Superposition:

- In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone.

## Superposition

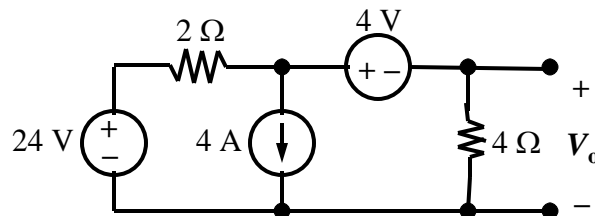
### Procedure:

1. Determine contribution due to **one** independent source
  - Set all other sources to 0: Replace independent voltage source by short circuit, independent current source by open circuit

**Do not turn independent sources off!!!**
2. Repeat for each independent source
3. Sum individual contributions to obtain desired voltage or current

## Superposition Example

- Find  $V_o$



## Equivalent Circuit Concept

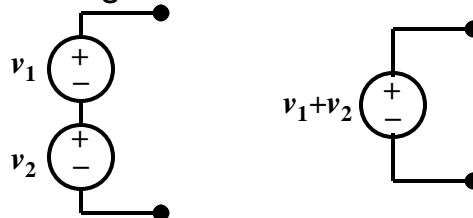
- A network of voltage sources, current sources, and resistors can be replaced by an **equivalent circuit** which has **identical terminal properties ( $I$ - $V$  characteristics)** without affecting the operation of the rest of the circuit.



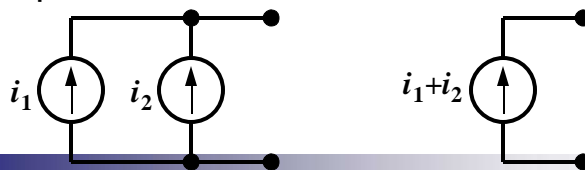
$$i_A(v_A) = i_B(v_B)$$

## Source Combinations

- Voltage sources in series can be replaced by an equivalent voltage source:

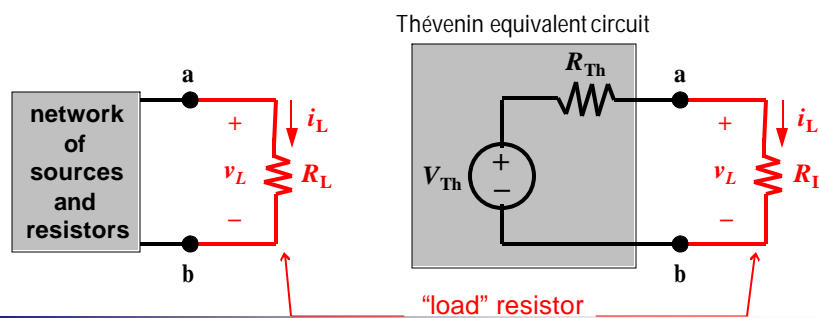


- Current sources in parallel can be replaced by an equivalent current source:



## Thévenin Equivalent Circuit

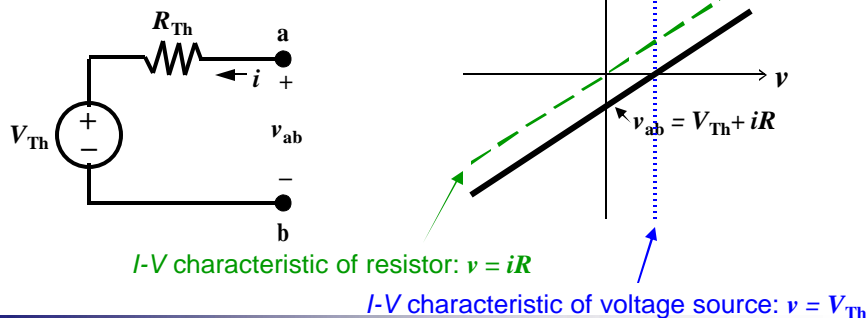
- Any\* *linear* 2-terminal (1-port) network of indep. voltage sources, indep. current sources, and linear resistors can be replaced by an equivalent circuit consisting of **an independent voltage source in series with a resistor** without affecting the operation of the rest of the circuit.



## I-V Characteristic of Thévenin Equivalent

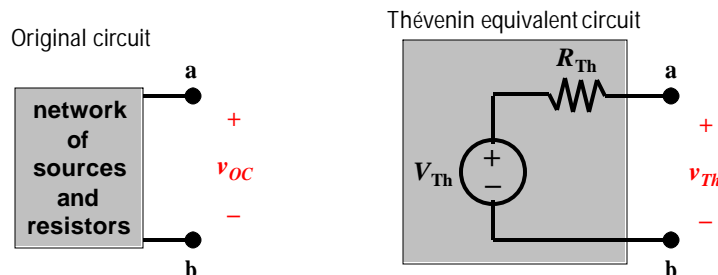
- The *I-V* characteristic for the series combination of elements is obtained by adding their voltage drops:

For a given current  $i$ , the voltage drop  $v_{ab}$  is equal to the sum of the voltages dropped across the source ( $V_{Th}$ ) and across the resistor ( $iR_{Th}$ )



## Procedure to Find $V_{Th}$

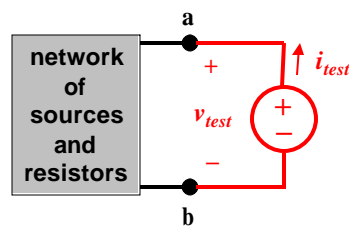
- Calculate the open circuit voltage  $v_{OC}$  from the original circuit.  $v_{OC} = V_{Th}$ .



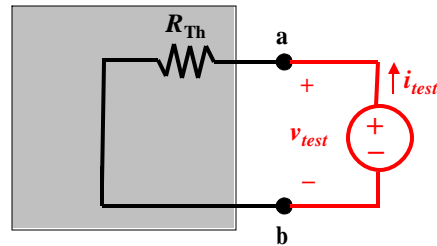
## Procedure to Find $R_{Th}$

1. Turn off all **independent** sources in original circuit. Dependent source should be left as is.
2. Apply a test voltage  $v_{test}$  source between terminals a and b. Calculate the resulting current  $i_{test}$ .
3.  $R_{th} = v_{test} / i_{test}$

Original circuit

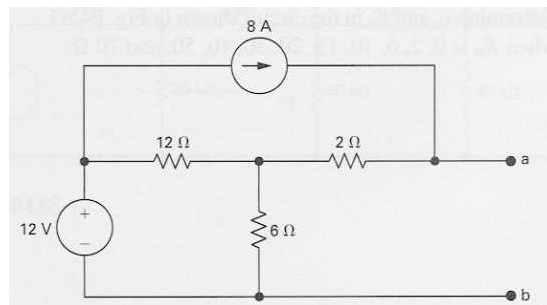


Thévenin equivalent circuit



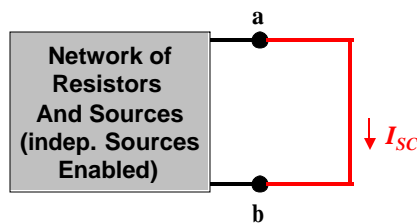
## Thévenin Equivalent Voltage Example

Find the Thevenin equivalent voltage with respect to the terminals a,b:

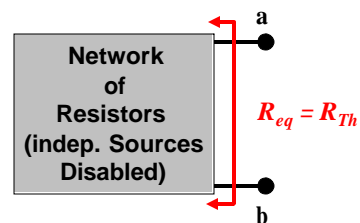


## More ways of calculating $R_{Th}$

1. Calculate  $I_{SC}$ , the short circuit current flowing from b to a.  $R_{Th} = V_{OC} / I_{SC}$
2. Turn off all independent sources and find the equivalent resistance  $R_{eq}$  between a and b.  $R_{Th} = R_{eq}$



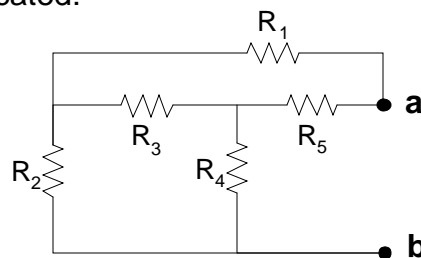
Method 1



Method 2

## Cautions when calculating $R_{Th}$ using algorithms from previous slide

- Method 1 is very quick, but cannot be used when  $I_{SC}$  or  $V_{Th} = 0$
- Method 2 cannot be used when there are dependent sources in the network or when the resistive network is too complicated:



- Advise: Use  $V_{test}$  and  $I_{test}$  method!!!



## Comments on Dependent Sources

A dependent source establishes a voltage or current whose value depends on the value of a voltage or current at a specified location in the circuit.

(device model, used to model behavior of transistors & amplifiers)

To specify a dependent source, we must identify:

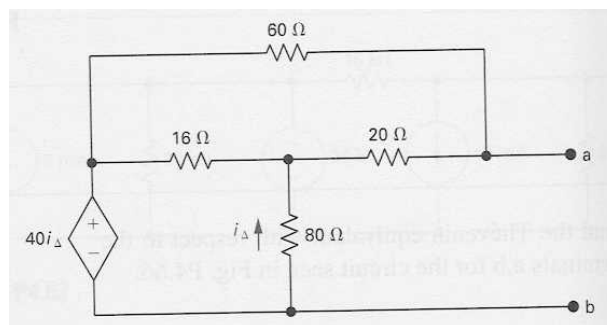
1. the controlling voltage or current (must be calculated, in general)
2. the relationship between the controlling voltage or current and the supplied voltage or current
3. the reference direction for the supplied voltage or current

**The relationship between the dependent source and its reference cannot be broken!**

- Dependent sources cannot be turned off for various purposes (e.g. to find the Thévenin resistance, or in analysis using Superposition).

## Thevenin Equivalent Example #2

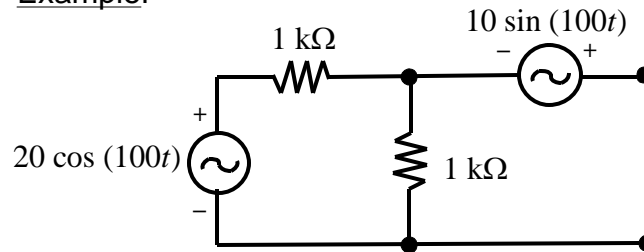
Find the Thevenin equivalent with respect to the terminals a,b:  
Methods 1 and 2 fail in this case.



## Networks Containing Time-Varying Sources

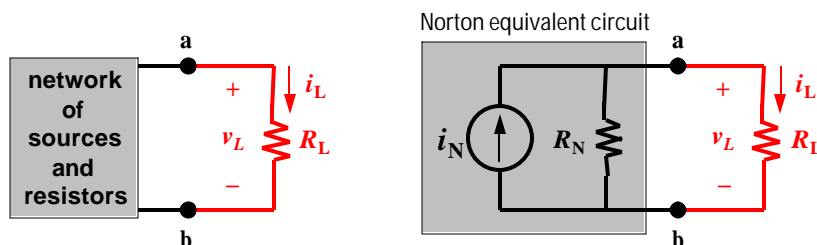
Care must be taken in summing time-varying sources!

Example:



## Norton Equivalent Circuit

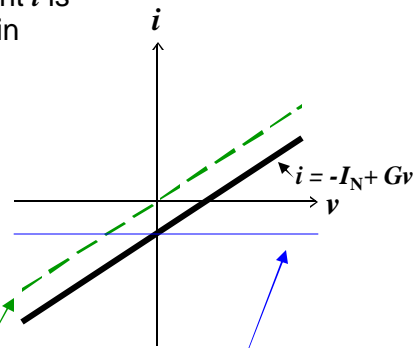
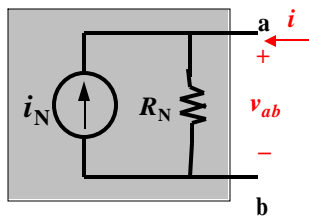
- Any\* *linear* 2-terminal (1-port) network of indep. voltage sources, indep. current sources, and linear resistors can be replaced by an equivalent circuit consisting of **an independent current source in parallel with a resistor** without affecting the operation of the rest of the circuit.



## I-V Characteristic of Norton Equivalent

- The I-V characteristic for the parallel combination of elements is obtained by adding their currents:

For a given voltage  $v_{ab}$ , the current  $i$  is equal to the sum of the currents in each of the two branches:



I-V characteristic of resistor:  $i = Gv$

I-V characteristic of current source:  $i = -I_N$

## Finding $I_N$ and $R_N = R_{Th}$

Analogous to calculation of Thevenin Eq. Ckt:

- 1) Find s.c. current and Norton (Thev) resistance

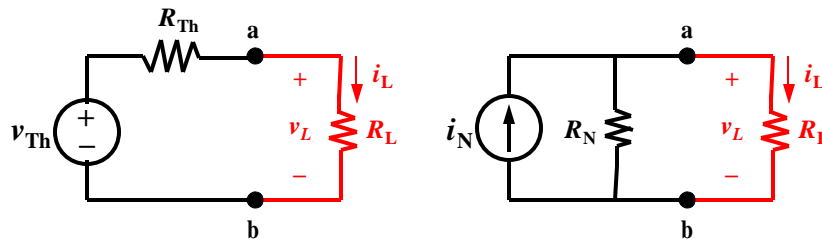
*Find  $I_{SC}$  and then use the  $V_{test} / I_{test}$  method to find  $R_N$*

- 2) or find o.c voltage and s.c. current (**with caution!!**)

$$I_N \quad I_{sc} = V_{Th} / R_{Th}$$

## Finding $I_N$ and $R_N$

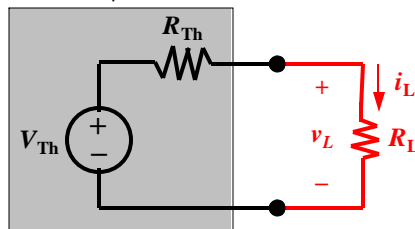
- We can derive the Norton equivalent circuit from a Thévenin equivalent circuit simply by making a source transformation:



$$R_N = R_{Th} = \frac{v_{oc}}{i_{sc}}; \quad i_N = \frac{v_{Th}}{R_{Th}} = i_{sc}$$

## Maximum Power Transfer Theorem

Thévenin equivalent circuit



Power absorbed by load resistor:

$$p = i_L^2 R_L = \left( \frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

To find the value of  $R_L$  for which  $p$  is maximum, set  $\frac{dp}{dR_L}$  to 0:

$$\frac{dp}{dR_L} = V_{Th}^2 \left[ \frac{(R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right] = 0$$

$$\Rightarrow (R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L) = 0$$

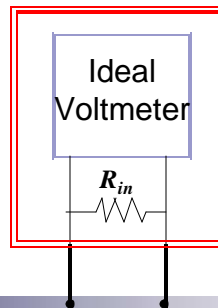
$$\Rightarrow R_{Th} = R_L$$

A resistive load receives maximum power from a circuit if the load resistance equals the Thévenin resistance of the circuit.

## Measuring Voltage

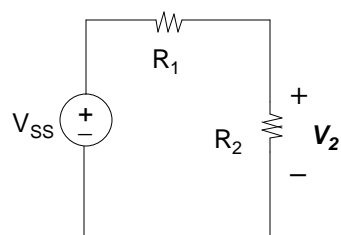
To measure the voltage drop across an element in a real circuit, insert a voltmeter (digital multimeter in voltage mode) **in parallel** with the element.

Voltmeters are characterized by their “voltmeter input resistance” ( $R_{in}$ ). Ideally, this should be very high (typical value 10 M $\Omega$ )



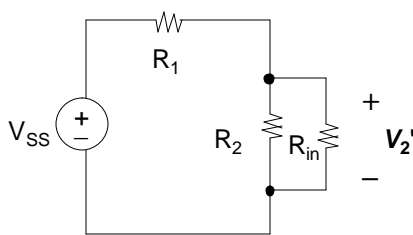
## Effect of Voltmeter

undisturbed circuit



$$V_2 = V_{SS} \left[ \frac{R_2}{R_1 + R_2} \right]$$

circuit with voltmeter inserted



$$V_2' = V_{SS} \left[ \frac{R_2 \parallel R_{in}}{R_2 \parallel R_{in} + R_1} \right]$$

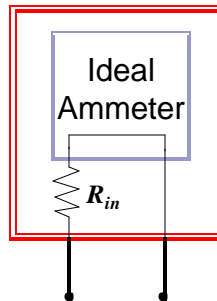
Example:  $V_{SS} = 10\text{ V}, R_2 = 100\text{ K}, R_1 = 900\text{ K} \Rightarrow V_2 = 1\text{ V}$

$$R_{in} = 10\text{ M}\Omega \Rightarrow V_2' = 0.9\text{ V}$$

## Measuring Current

To measure the current flowing through an element in a real circuit, insert an ammeter (digital multimeter in current mode) **in series** with the element.

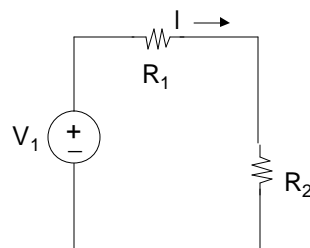
Ammeters are characterized by their “ammeter input resistance” ( $R_{in}$ ). Ideally, this should be very low (typical value  $1\Omega$ ).



## Effect of Ammeter

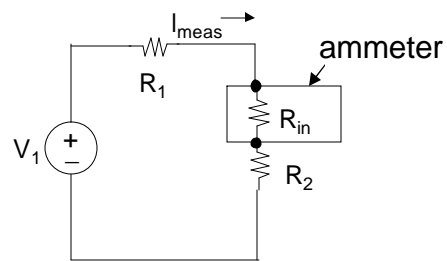
Measurement error due to non-zero input resistance:

undisturbed circuit



$$I = \frac{V_1}{R_1 + R_2}$$

circuit with ammeter inserted



$$I_{\text{meas}} = \frac{V_1}{R_1 + R_2 + R_{in}}$$

Example:  $V_1 = 1\text{ V}$ ,  $R_1 = R_2 = 500\ \Omega$ ,  $R_{in} = 1\ \Omega$

$$I = \frac{1\text{ V}}{500\ \Omega + 500\ \Omega} = 1\text{ mA}, \quad I_{\text{meas}} = \frac{1\text{ V}}{500\ \Omega + 500\ \Omega + 1\ \Omega} \approx 1\text{ mA}$$