Announcements

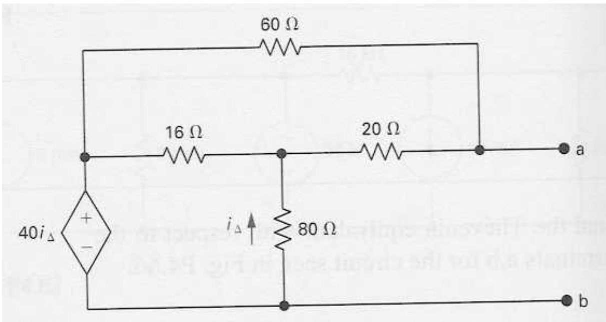
- HW #1 Due today at 6pm.
- HW #2 posted online today and due next Tuesday at 6pm.
- No labs or discussions tomorrow. If you want to attend discussion, go on Friday.
- Midterm tentatively Tuesday 7/11.

Review

- Mesh and Nodal Analysis
- Superposition
- Equivalent Circuits
  - Thevenin
  - Norton
- Measuring Voltages and Currents

Review: Thevenin Equivalent Example

Find the Thevenin equivalent with respect to the terminals a,b:



Lecture #4

OUTLINE

- The capacitor
- The inductor
- 1st Order Circuits
- Transient and Steady-State response

Reading

Chapter 3, Chap 4.1-4.5

The Capacitor

Two conductors (a,b) separated by an insulator:

difference in potential = V_{ab}

=> equal & opposite charge Q on conductors

$$\boxed{Q = CV_{ab}}$$ (stored charge in terms of voltage)

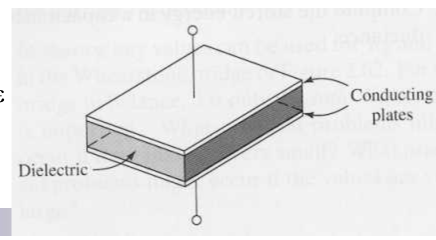
where C is the **capacitance** of the structure,

➢ positive (+) charge is on the conductor at higher potential

Parallel-plate capacitor:

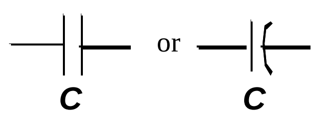
- area of the plates = A ***(m²)***
- separation between plates = d ***(m)***
- ***dielectric permittivity*** of insulator = ε ***(F/m)***

=> capacitance $\boxed{C = \frac{A\varepsilon}{d}}$ ***F(F)***



Capacitor

Symbol:



\+ C

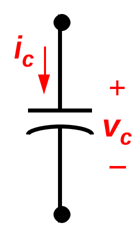
Electrolytic (polarized) capacitor

Units: Farads (Coulombs/Volt)

Current-Voltage relationship:

$$i_c = \frac{dQ}{dt} = C\frac{dv_c}{dt} + v_c\frac{dC}{dt}$$

If C (geometry) is unchanging, $i_C = C\, dv_C/dt$



Note: Q (v_c) must be a continuous function of time

Voltage in Terms of Current

$$Q(t) = \int_0^t i_c(t)dt + Q(0)$$

$$v_c(t) = \frac{1}{C}\int_0^t i_c(t)dt + \frac{Q(0)}{C} = \frac{1}{C}\int_0^t i_c(t)dt + v_c(0)$$

Uses: Capacitors are used to store energy for camera flashbulbs, in filters that separate various frequency signals, and they appear as undesired "parasitic" elements in circuits where they usually degrade circuit performance

Stored Energy

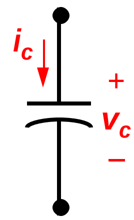
CAPACITORS STORE ELECTRIC ENERGY

You might think the energy stored on a capacitor is ***QV* = *CV*²**, which has the dimension of Joules. But during charging, the average voltage across the capacitor was only half the final value of ***V*** for a linear capacitor.

Thus, energy is $\frac{1}{2}QV = \boxed{\frac{1}{2}CV^2}$.

Example: A 1 pF capacitance charged to 5 Volts has ½(5V)² (1pF) = 12.5 pJ

A more rigorous derivation

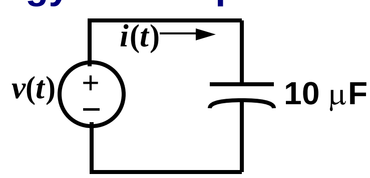


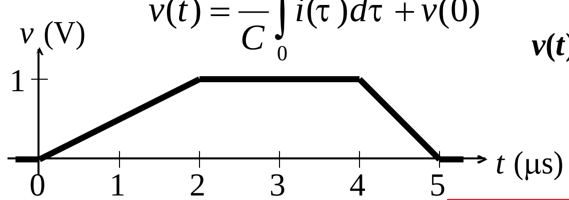
$$w = \int_{t=t_{Initial}}^{t=t_{Final}} v_c \cdot i_c \, dt = \int_{v=V_{Initial}}^{v=V_{Final}} v_c \frac{dQ}{dt} dt = \int_{v=V_{Initial}}^{v=V_{Final}} v_c \, dQ$$

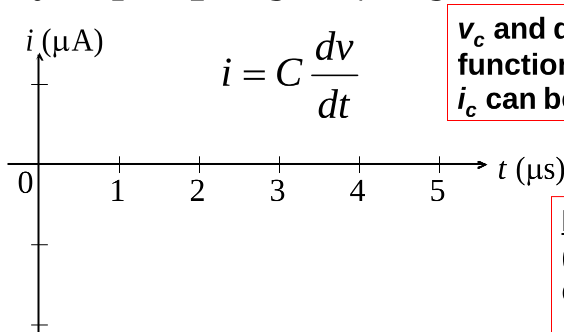
$$w = \int_{v=V_{Initial}}^{v=V_{Final}} C v_c \, dv_c = \frac{1}{2} C V_{Final}{}^2 - \frac{1}{2} C V_{Initial}{}^2$$

Example: Current, Power & Energy for a Capacitor

$$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + v(0)$$





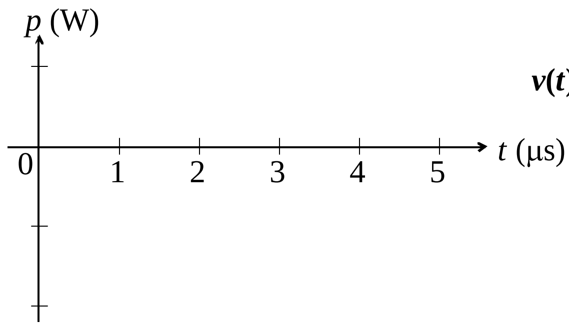


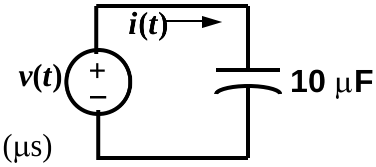
$$i = C\frac{dv}{dt}$$

v_c and q must be continuous functions of time; however, i_c can be discontinuous.

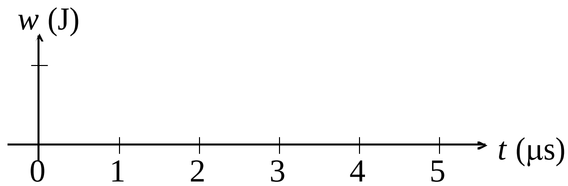
<u>Note</u>: In "steady state" (dc operation), time derivatives are zero → *C* is an open circuit

Example: Current, Power & Energy for a Capacitor



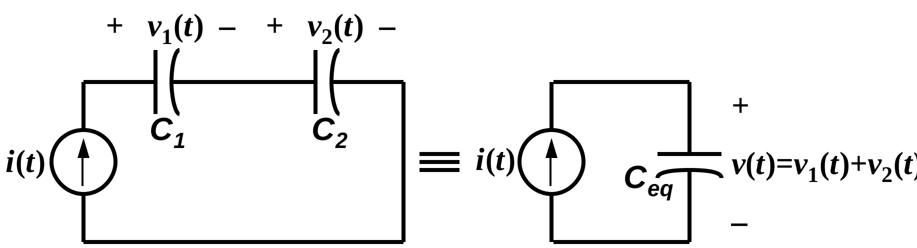


$$p = vi$$



$$w = \int_0^t p\,d\tau = \frac{1}{2}Cv^2$$

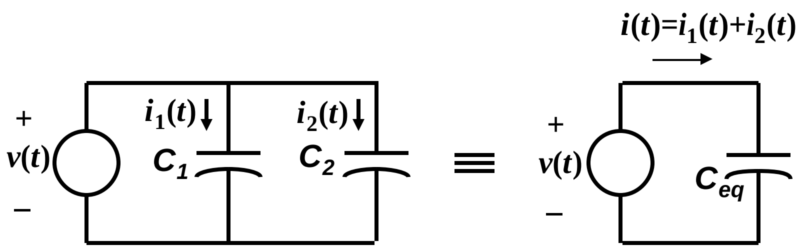
Capacitors in Series



$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Proof:

Capacitors in Parallel

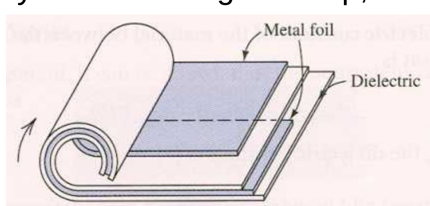


$$\boxed{C_{eq} = C_1 + C_2}$$

Proof:

Practical Capacitors

- A capacitor can be constructed by interleaving the plates with two dielectric layers and rolling them up, to achieve a compact size.



- To achieve a small volume, a very thin dielectric with a high dielectric constant is desirable. However, dielectric materials break down and become conductors when the electric field (units: V/cm) is too high.
  - Real capacitors have maximum voltage ratings
  - An engineering trade-off exists between compact size and high voltage rating

Inductor

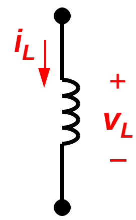
Symbol:

Units: Henrys (Volts • second / Ampere)

Current in terms of voltage:

$$di_L = \frac{1}{L} v_L(t)dt$$

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(\tau)d\tau + i(t_0)$$



Note: i_L **must be a continuous function of time**

Stored Energy

INDUCTORS STORE MAGNETIC ENERGY

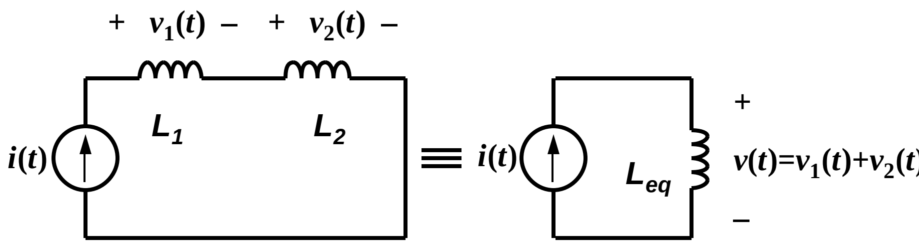
Consider an inductor having an initial current $i(t_0) = i_0$

$$p(t) = v(t)i(t) =$$

$$w(t) = \int_{t_0}^{t} p(\tau)d\tau =$$

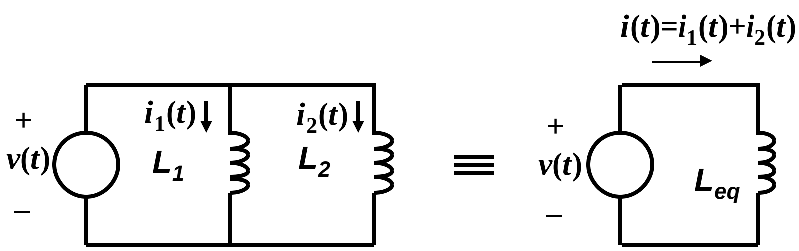
$$w(t) = \frac{1}{2}Li^2 - \frac{1}{2}Li_0{}^2$$

Inductors in Series



$$\boxed{L_{eq} = L_1 + L_2}$$

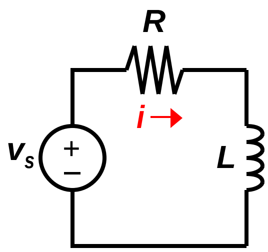
Inductors in Parallel

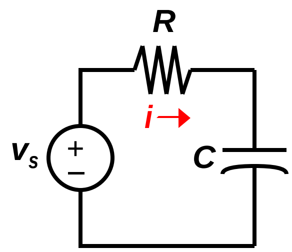


$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

First-Order Circuits

- A circuit that contains only sources, resistors and an inductor is called an ***RL circuit***.
- A circuit that contains only sources, resistors and a capacitor is called an ***RC circuit***.
- RL and RC circuits are called first-order circuits because their voltages and currents are described by first-order differential equations.



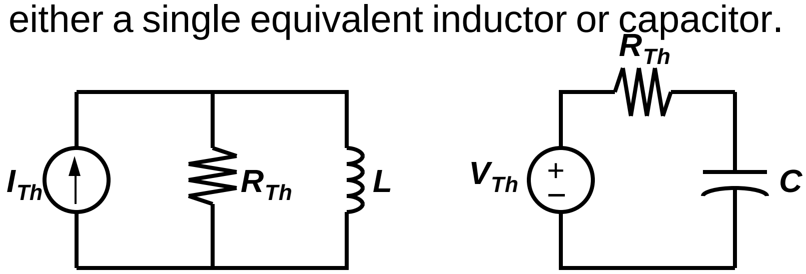


Transient *vs.* Steady-State Response

- The momentary behavior of a circuit (in response to a change in stimulation) is referred to as its ***transient response***.
- The behavior of a circuit a long time (many time constants) after the change in voltage or current is called the ***steady-state response***.

Review (Conceptual)

- Any* first-order circuit can be reduced to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor.



  - In steady state, an inductor behaves like a short circuit
  - In steady state, a capacitor behaves like an open circuit

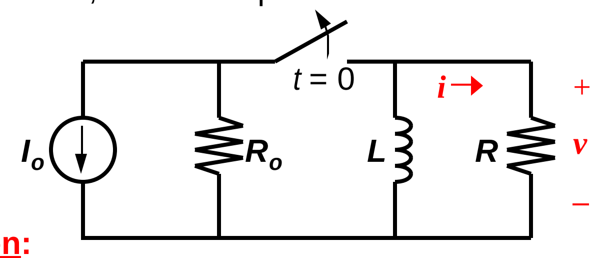
Response

- The ***natural response*** of an RL or RC circuit is its behavior (*i.e.,* current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing no independent sources).

- The ***step response*** of an RL or RC circuit is its behavior when a voltage or current source **step** is applied to the circuit, or immediately after a switch state is changed.

Natural Response of an RL Circuit

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:



Notation:

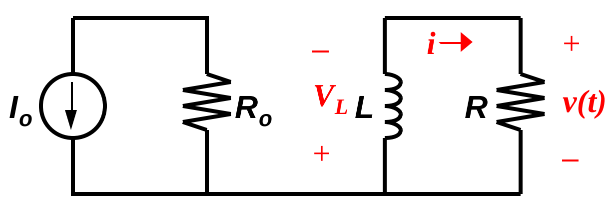
0^- is used to denote the time just prior to switching

0^+ is used to denote the time immediately after switching

- t<0 the entire system is at steady-state; and the inductor is → like short circuit
- The current flowing in the inductor at $t = 0^-$ is I_o and $v = 0$

Solving for the Current ($t \geq 0$)

- For t > 0, the circuit reduces to



- $v(t) = i(t)R$
- At $t = 0^+$: $i(0^-) = i(0^+) = I_o$ since the current is continuous in L

Solving for the Current ($t \geq 0$)

- Applying KVL to the LR circuit:

$$V_L = -v(t)$$

$$L\frac{di(t)}{dt} = -i(t)R \quad \Rightarrow \quad i(t) + \frac{L}{R}\frac{di(t)}{dt} = 0$$

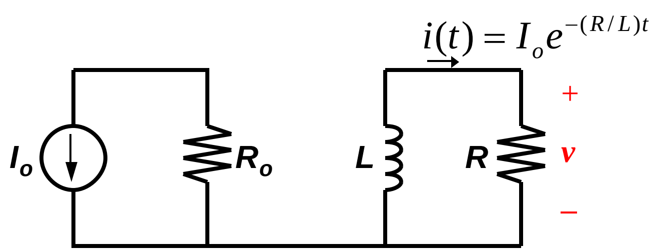
- The solution to this ODE equation is:

$$i(t) = Ae^{-\frac{t}{\tau}} \quad where\ \tau = \frac{L}{R}$$

- Finding A:

$$i(0^+) = I_o \quad \Rightarrow \quad i(t) = I_o e^{-\frac{R}{L}t}$$

Solving for the Voltage ($t > 0$)



- Note that the voltage changes abruptly:

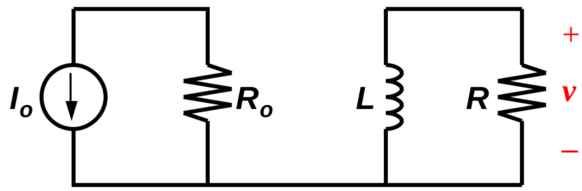
$$v(0^-) = 0$$

$$\text{for } t > 0,\ v(t) = iR = I_o R e^{-(R/L)t}$$

$$\Rightarrow v(0^+) = I_0 R$$

Solving for Power and Energy Delivered (t > 0)

$$i(t) = I_o e^{-(R/L)t}$$



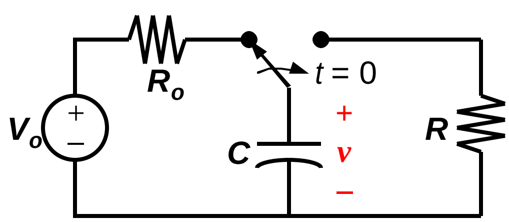
$$p = i^2 R = I_o^2 R e^{-2(R/L)t}$$

$$w = \int_0^t p(x)dx = \int_0^t I_o^2 R e^{-2(R/L)x} dx$$

$$= \frac{1}{2} L I_o^2 \left(1 - e^{-2(R/L)t}\right)$$

Natural Response of an RC Circuit

- Consider the following circuit, for which the switch is closed for t < 0, and then opened at t = 0:



Notation:

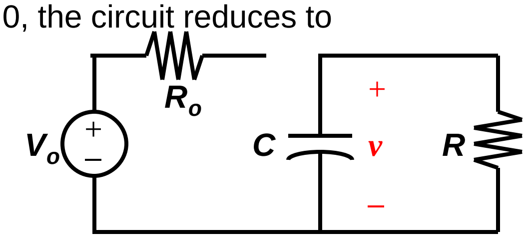
0^- is used to denote the time just prior to switching

0^+ is used to denote the time immediately after switching

- The voltage on the capacitor at $t = 0^-$ is V_o. ***Why?***

Solving for the Voltage ($t \geq 0$)

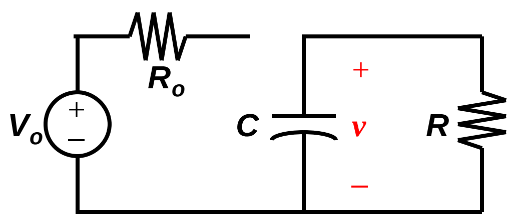
- For t > 0, the circuit reduces to



- Applying KCL to the RC circuit:

- Solution: $v(t) = v(0)e^{-t/RC}$

Solving for the Current ($t > 0$)



$$v(t) = V_o e^{-t/RC}$$

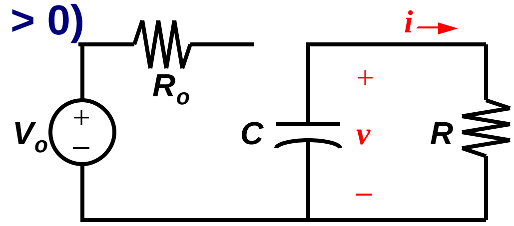
- Note that the current changes abruptly:

$$i(0^-) = 0$$

$$\text{for } \; t > 0, \; i(t) = \frac{v}{R} = \frac{V_o}{R} e^{-t/RC}$$

$$\Rightarrow \; i(0^+) = \frac{V_o}{R}$$

Solving for Power and Energy Delivered (t > 0)



$$v(t) = V_o e^{-t/RC}$$

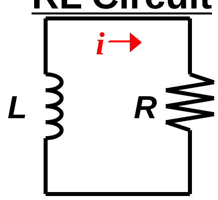
$$p = \frac{v^2}{R} = \frac{V_o^2}{R} e^{-2t/RC}$$

$$w = \int_0^t p(x)dx = \int_0^t \frac{V_o^2}{R} e^{-2x/RC} dx$$

$$= \frac{1}{2} C V_o^2 \left(1 - e^{-2t/RC}\right)$$

Natural Response Summary

RL Circuit



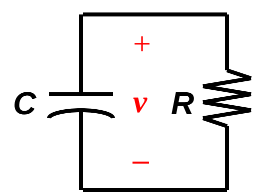
- Inductor current cannot change instantaneously

$$i(0^-) = i(0^+)$$

$$i(t) = i(0) e^{-t/\tau}$$

- time constant $\tau = \dfrac{L}{R}$

RC Circuit



- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0) e^{-t/\tau}$$

- time constant $\tau = RC$

Procedure for Finding Transient Response

1. **Identify the variable of interest**
   - For RL circuits, it is usually the inductor current $i_L(t)$
   - For RC circuits, it is usually the capacitor voltage $v_c(t)$
2. **Determine the initial value (at $t = t_0^+$) of the variable**
   - Recall that $i_L(t)$ and $v_c(t)$ are continuous variables:

     $i_L(t_0^+) = i_L(t_0^-)$ and $v_c(t_0^+) = v_c(t_0^-)$
   - Assuming that the circuit reached steady state before t_0, use the fact that an inductor behaves like a short circuit in steady state or that a capacitor behaves like an open circuit in steady state

Procedure (cont'd)

3. **Calculate the final value of the variable**
   (its value as $t \rightarrow$)
   - Again, make use of the fact that an inductor behaves like a short circuit in steady state ($t \rightarrow$) or that a capacitor behaves like an open circuit in steady state ($t \rightarrow$)
4. **Calculate the time constant for the circuit**

   $\tau = L/R$ for an RL circuit, where R is the Thévenin equivalent resistance "seen" by the inductor

   $\tau = RC$ for an RC circuit where R is the Thévenin equivalent resistance "seen" by the capacitor

Summary

Capacitor

$$i = C\frac{dv}{dt}$$

$$w = \frac{1}{2}Cv^2$$

v cannot change instantaneously

i can change instantaneously

Do not short-circuit a charged capacitor (-> infinite current!)

n cap.'s in series: $\frac{1}{C_{eq}} = \sum_{i=1}^{n}\frac{1}{C_i}$

n cap.'s in parallel: $C_{eq} = \sum_{i=1}^{n}C_i$

Inductor

$$v = L\frac{di}{dt}$$

$$w = \frac{1}{2}Li^2$$

i cannot change instantaneously

v can change instantaneously

Do not open-circuit an inductor with current (-> infinite voltage!)

n ind.'s in series: $L_{eq} = \sum_{i=1}^{n}L_i$

n ind.'s in parallel: $\frac{1}{L_{eq}} = \sum_{i=1}^{n}\frac{1}{L_i}$

Summary Cont'd

- Steady-state → nothing is time varying.
- In steady state, an inductor behaves like a short circuit
- In steady state, a capacitor behaves like an open circuit