Review

- Phasors
  - Source vs. Impedence representation

- First Order Circuits
  - Initial and Final conditions

- Second Order Circuits
  - Solution

Lecture #7

OUTLINE

- Decibels
- Transfer function
- First-order lowpass filter
- Cascade connection and Logarithmic frequency scales
- Bode Plots

Reading

- Chap 6

Bel and Decibel (dB)

- A **bel** (symbol **B**) is a unit of measure of ratios of power levels, i.e. relative power levels.
  - The name was coined in the early 20th century in honor of Alexander Graham Bell, a telecommunications pioneer.
  - The bel is a logarithmic measure. The number of bels for a given ratio of power levels is calculated by taking the logarithm, to the base 10, of the ratio.
  - one bel corresponds to a ratio of 10:1.
  - $B = \log_{10}(P_1/P_2)$ where P_1 and P_2 are power levels.
- The bel is too large for everyday use, so the **decibel (dB)**, equal to 0.1B, is more commonly used.
  - $1\text{dB} = 10 \log_{10}(P_1/P_2)$
- dB are used to measure
  - Electric power, Gain or loss of amplifiers, Insertion loss of filters.

Logarithmic Measure for Power

- To express a power in terms of decibels, one starts by choosing a reference power, $P_{reference}$, and writing

  Power P in decibels = $10 \log_{10}(P/P_{reference})$
- Exercise:
  - Express a power of 50 mW in decibels relative to 1 watt.
  - $P\,(\text{dB}) = 10 \log_{10}(50 \times 10^{-3}) = -13$ dB
- Exercise:
  - Express a power of 50 mW in decibels relative to 1 mW.
  - $P\,(\text{dB}) = 10 \log_{10}(50) = 17$ dB.
- dBm to express absolute values of power relative to a milliwatt.
  - $\text{dBm} = 10 \log_{10}$ (power in milliwatts / 1 milliwatt)
  - 100 mW = 20 dBm
  - 10 mW = 10 dBm

Power in 1st and 2nd Order Circuits

- **When dealing with 1st or 2nd order cirduits it is convenient to refer to the frequency difference between points at which the power from the circuit is half that at the peak of resonance.**
- **Such frequencies are known as "half-power frequencies", and the power output there referred to the peak power (at the resonant frequency) is**
- **$10\log_{10}(P_{half\text{-}power}/P_{peak}) = 10\log_{10}(1/2) = -3$ dB.**

Logarithmic Measures for Voltage or Current

From the expression for power ratios in decibels, we can readily derive the corresponding expressions for voltage or current ratios.

Suppose that the voltage *V* (or current *I*) appears across (or flows in) a resistor whose resistance is *R*. The corresponding power dissipated, *P*, is V^2/R (or I^2R). We can similarly relate the reference voltage or current to the reference power, as

$$P_{reference} = (V_{reference})^2/R \text{ or } P_{reference} = (I_{reference})^2R.$$

Hence,

Voltage, V in decibels = $20\log_{10}(V/V_{reference})$

Current, I, in decibels = $20\log_{10}(I/I_{reference})$

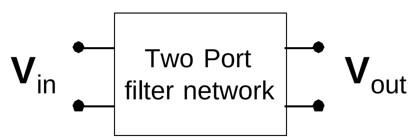
Note that the voltage and current expressions are just like the power expression except that they have 20 as the multiplier instead of 10 because power is proportional to the square of the voltage or current.

Exercise: How many decibels larger is the voltage of a 9-volt transistor battery than that of a 1.5-volt AA battery? Let $V_{reference}$ = 1.5. The ratio in decibels is

$$20 \log_{10}(9/1.5) = 20 \log_{10}(6) = 16 \text{ dB.}$$

Transfer Function

- Transfer function is a function of frequency
  - Complex quantity
  - Both magnitude and phase are function of frequency

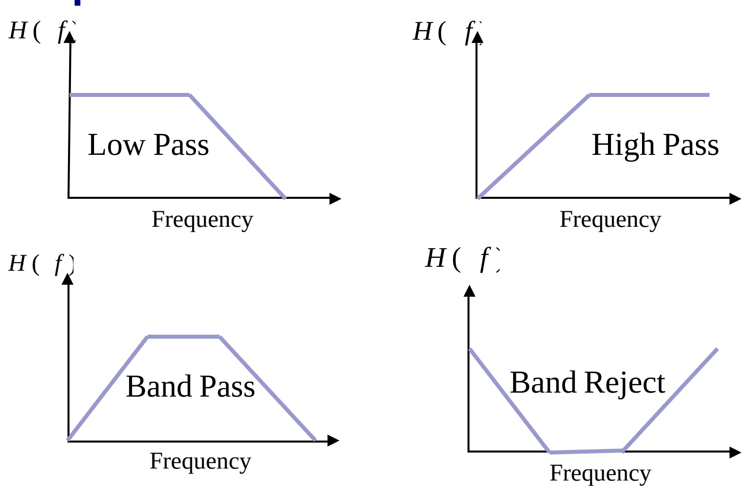


$$H(\omega) = \frac{V_{out}}{V_{in}} \angle(\theta_{out} - \theta_{in})$$

Filters

- Circuit designed to retain a certain frequency range and discard others
  - *Low-pass*: pass low frequencies and reject high frequencies
  - *High-pass*: pass high frequencies and reject low frequencies
  - *Band-pass*: pass some particular range of frequencies, reject other frequencies outside that band
  - *Notch*: reject a range of frequencies and pass all other frequencies

Common Filter Transfer Function vs. Freq

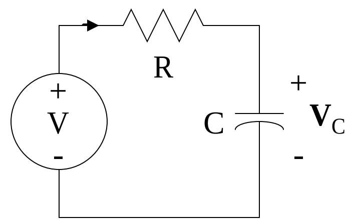


First-Order Lowpass Filter

$$H(j\omega) = \frac{V_C}{V} = \frac{1/(j\omega C)}{1/(j\omega C)+R} = \frac{1}{1+j\omega RC} = \frac{1}{\sqrt{1+(\omega RC)^2}} \angle -\tan^{-1}(\omega RC)$$

$$Let\,\omega_B = 1/RC \quad and \quad f_B = 1/2\pi RC$$

$$H(j\omega) = \frac{1}{1+j\dfrac{\omega}{\omega_{\mathbf{B}}}} = \frac{1}{\sqrt{1+(\dfrac{\omega}{\omega_{\mathbf{B}}})^2}} \angle -\tan^{-1}(\frac{\omega}{\omega_{\mathbf{B}}})$$



$$H(j\omega_B) = \frac{1}{\sqrt{2}} \angle -\tan^{-1}(1)$$

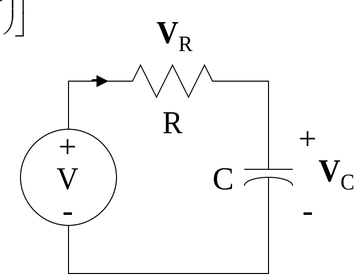
$$20\log_{10}\left(\frac{|H(j\omega_B)|}{|H(0)|}\right) = 20\log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3dB$$

First-Order Highpass Filter

$$H(j\omega) = \frac{V_R}{V} = \frac{R}{1/(j\omega C)+R} = \frac{j\omega RC}{1+j\omega RC} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \angle -\left[\frac{\pi}{2} - \tan^{-1}(\omega RC)\right]$$

$$Let\,\omega_B = 1/RC \quad and \quad f_B = 1/2\pi RC$$

$$H(j\omega) = \frac{\dfrac{\omega}{\omega_{\mathbf{B}}}}{1+j\dfrac{\omega}{\omega_{\mathbf{B}}}} = \frac{\dfrac{\omega}{\omega_{\mathbf{B}}}}{\sqrt{1+(\dfrac{\omega}{\omega_{\mathbf{B}}})^2}} \angle -\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{\omega_{\mathbf{B}}}\right)\right]$$

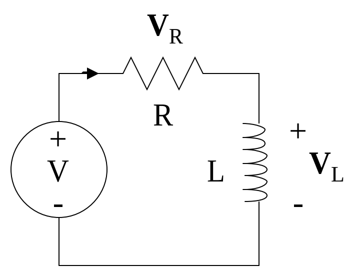


$$H(j\omega_B) = \frac{1}{\sqrt{2}} \angle -\tan^{-1}(1)$$

$$20\log_{10}\left(\frac{|H(j\omega_B)|}{|H(0)|}\right) = 20\log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3dB$$

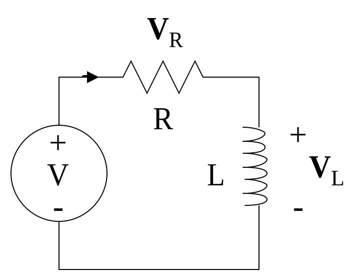
First-Order Lowpass Filter

$$H(j\omega) = \frac{V_R}{V} = \ldots$$

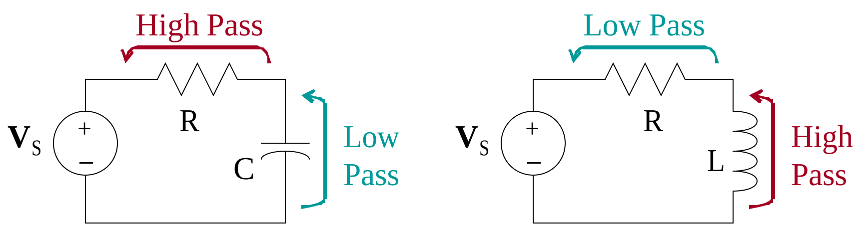


First-Order Highpass Filter

$$H(j\omega) = \frac{V_L}{V} = \ldots$$



First-Order Filter Circuits



$\mathbf{H}_R = R / (R + 1/j\omega C)$ $\quad$ $\mathbf{H}_R = R / (R + j\omega L)$

$\mathbf{H}_C = (1/j\omega C) / (R + 1/j\omega C)$ $\quad$ $\mathbf{H}_L = j\omega L / (R + j\omega L)$

Gain or Loss Expressed in Decibels

The gain produced by an amplifier or the loss of a filter is often specified in decibels.

The input voltage (current, or power) is taken as the reference value of voltage (current, or power) in the decibel defining expression:

Voltage gain in dB = **20** $\log_{10}(V_{output}/V_{input})$
Current gain in dB = **20** $\log_{10}(I_{output}/I_{input}$
Power gain in dB = **10** $\log_{10}(P_{output}/P_{input})$

Example: The voltage gain of an amplifier whose input is 0.2 mV and whose output is 0.5 V is

$$20\log_{10}(0.5/0.2\times10^{-3}) = 68 \text{ dB}.$$

Change of Voltage or Current with a Change of Frequency

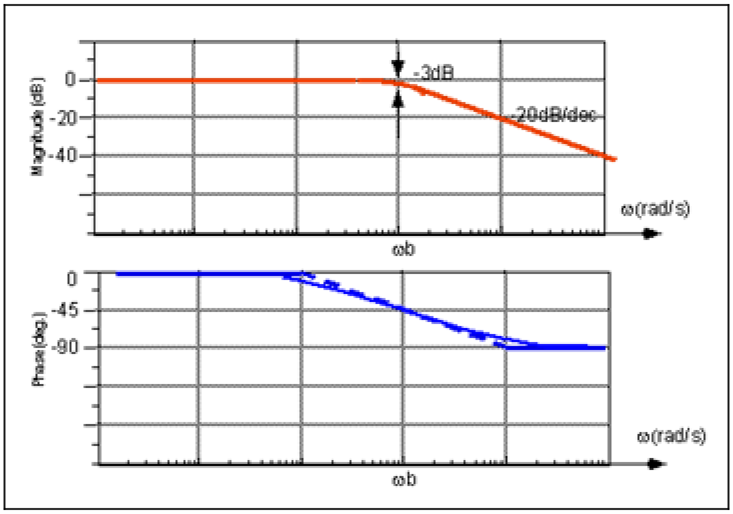
One may wish to specify the change of a quantity such as the output voltage of a filter when the frequency changes by a factor of 2 (an octave) or 10 (a decade).

For example, a single-stage RC low-pass filter has at frequencies above $\omega = 1/RC$ an output that changes at the rate -20dB per decade.

Bode Plot

- Plot of magnitude of transfer function vs. frequency
  - y-axis is the 20log of the magnitude of the transfer function in dB and x-axis is ω
  - Both x and y axes are in log scale
- Plot of phase of transfer function vs. frequency
  - y-axis is the phase of transfer function in degrees and x-axis is ω
  - y-axis is linear and x-axis is in log scale

Bode Plot Example



Magnitude Plot of a Low Pass Filter

$$H(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega}{\omega_{\mathbf{B}}})^2}} \angle -\tan^{-1}(\frac{\omega}{\omega_{\mathbf{B}}})$$

Note that :

$$|H(0)| = 1 \quad and \quad \lim_{\omega \to \infty}(|H(j\omega)|) = 0$$

$$|H(j\omega_B)| = \frac{1}{\sqrt{2}}$$

$$For\ \omega >> \omega_B, \quad |H(j\omega)| \approx \frac{\omega_B}{\omega}$$

$$For\ \omega << \omega_B, \quad |H(j\omega)| \approx 1$$

Magnitude Plot…

Remember we are plotting 20log(|H(jω)|)

$$20\log(|H(j\omega_B)|) = -3dB$$

$$For\ \omega << \omega_B, \quad |H(j\omega)| \approx 1$$

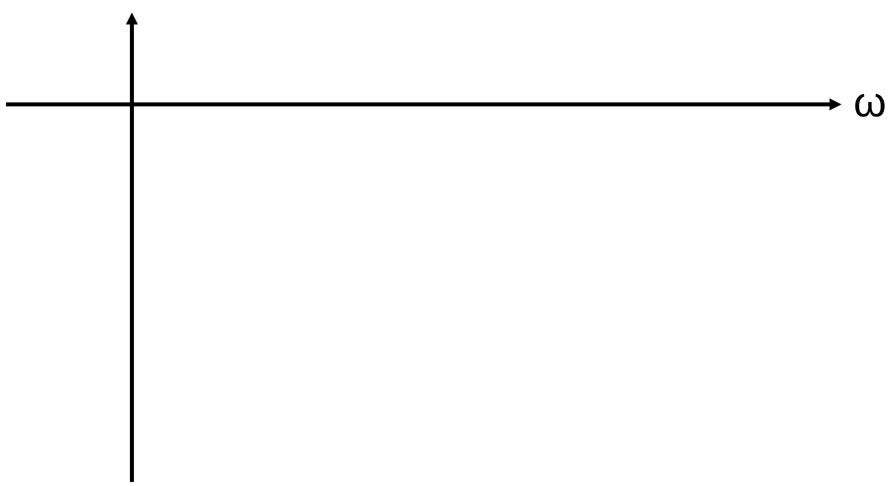
$$so \quad 20\log(|H(j\omega)|) = 0dB$$

$$For\ \omega >> \omega_B, \quad |H(j\omega)| \approx \frac{\omega_B}{\omega}$$

$$so \quad 20\log(|H(j\omega)|) = \underbrace{20\log(\omega_B)}_{\text{constant}} - \underbrace{20\log(\omega)}_{\text{Corresponds to a slope of -20dB/decade}}$$

Magnitude Plot

20log(|H(jω)|)



Phase Plot of a Low Pass Filter

$$H(j\omega) = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_B}\right)^2}} \angle - \tan^{-1}\left(\frac{\omega}{\omega_B}\right)$$

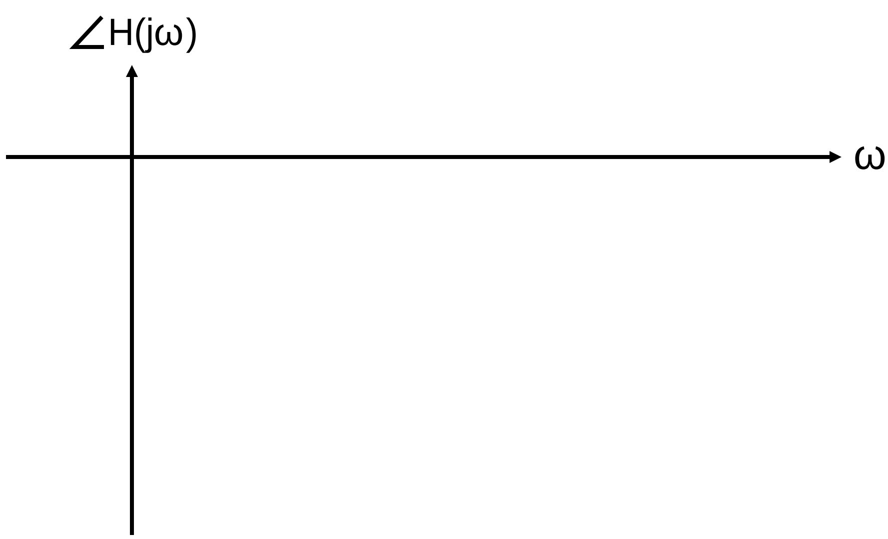
$$\textit{Note that:}$$

$$\angle H(0) = 0^o$$

$$\angle H(j\omega_B) = -45^o$$

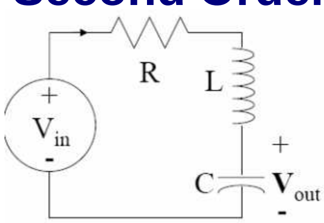
$$\lim_{\omega \to \infty} \angle H(j\omega) = -90^o$$

Magnitude Plot



Bode Plot of High Pass Filter…

Second Order Circuits



$$\mathbf{H(f)} = \frac{\mathbf{V}_c}{\mathbf{V}} = \frac{1/(j\omega C)}{1/(j\omega C)+R+j\omega L} = \frac{1}{1-\omega^2 LC + j\omega RC}$$

$$= \frac{1}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}} \angle\left[-\tan^{-1}\left(\frac{\omega RC}{1-\omega^2 LC}\right)\right]$$

In plotting the magnitude Bode plot, we should evaluate the extreme cases: $\omega=0$ and infinity

In addition, we should examine 0+ (small positive), and the frequency at which $|\mathbf{H(f)}|$ is maximum or minimum.

Examine $|\mathbf{H(f)}|$, we can easily see that max occurs at $\omega_0 = \left(\frac{1}{\sqrt{LC}}\right)$

$$\max|\mathbf{H(f)}| = \left(\frac{1}{\omega_0 RC}\right) = \frac{\sqrt{LC}}{RC}$$

Also remember that $\frac{R}{2L}\sqrt{C}$

Magnitude Response For $\zeta < 1$

$$H(j\omega) = \frac{1}{1-\omega^2 LC + j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{\left(1-\omega^2 LC\right)^2 + \left(\omega RC\right)^2}} = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_o^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_o}\right)^2}}$$

$$For\ \omega >> \omega_o, \quad |H(j\omega)| \approx \frac{1}{\omega^2 LC} = \frac{\omega_o^2}{\omega^2}$$

$$so \quad 20\log(|H(j\omega)|) = 40\log(\omega_B) - 40\log(\omega)$$

This corresponds to a slope of -40dB/decade

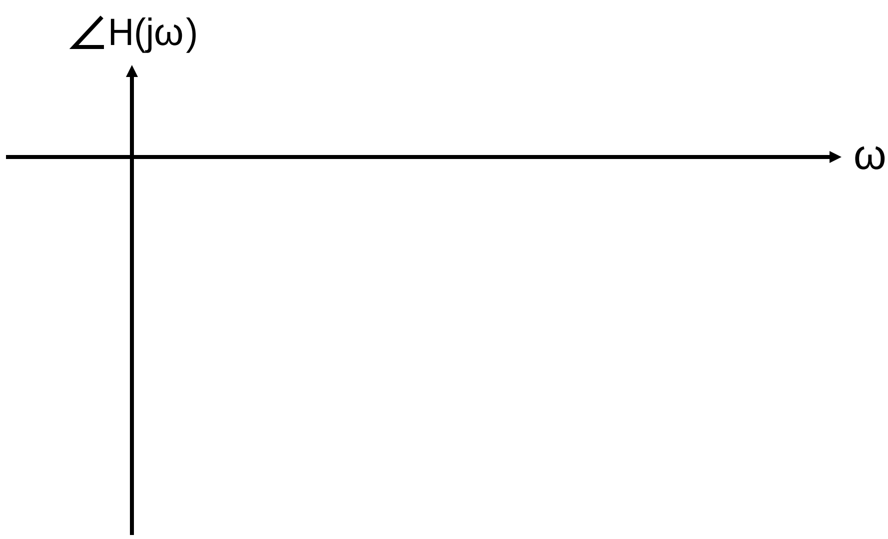
$$For\ \omega << \omega_o, \quad |H(j\omega)| \approx 1$$

$$so \quad 20\log(|H(j\omega)|) = 0dB$$

Note that:

$$|H(j\omega_o)| = \frac{1}{2\zeta}$$

Magnitude Plot For $\zeta < 1$



Phase Response For ζ < 1

$$For\ \omega >> \omega_o, \quad |H(\omega)| \approx \frac{1}{\omega^2 LC} = \frac{\omega_o^{\,2}}{\omega^2}$$

$$For\ \omega << \omega_o, \quad |H(\omega)| \approx 1$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega RC}{1-\omega^2 LC}\right) = -\tan^{-1}\left(\frac{2\zeta \dfrac{\omega}{\omega_o}}{1-\dfrac{\omega^2}{\omega_o^{\,2}}}\right)$$

Note that :

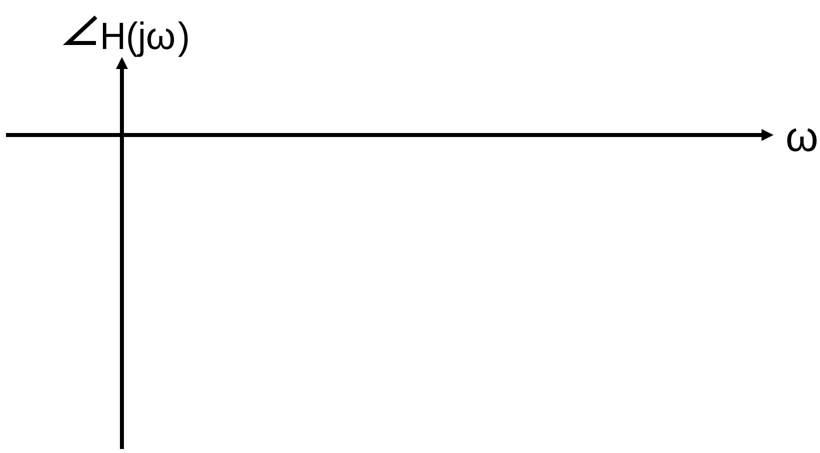
$$\angle H(0) = 0^o \quad and \quad \lim_{\omega \to \infty}(\angle H(j\omega)) = -180^o$$

$$\angle H(j\omega_o) = -90^o$$

Approximations for $\zeta \leq 1$:

$$\angle H(j0.1\omega_o) \cong 0^o \quad and \quad \angle H(j10\omega_o) \cong -180^o$$

Phase Response For ζ < 1 Continued



Bode Plot For ζ >= 1

- For ζ >= 1, we can know that we factor H(jω) in the following form:

$$H(j\omega) = \frac{1}{\left(1+\frac{\omega}{\omega_1}\right)\left(1+\frac{\omega}{\omega_2}\right)}$$

where $\omega_1 = \omega_2$ if ζ = 1

- We shall find the bode plot of this transfer function when we talk about higher order filters.

Higher Order Filters

- Higher order filters can be factorized into a product of 1st and 2nd order filters
- How do we account for the complete response?
  - Simply add all individual responses to get the overall response

Addition of responses

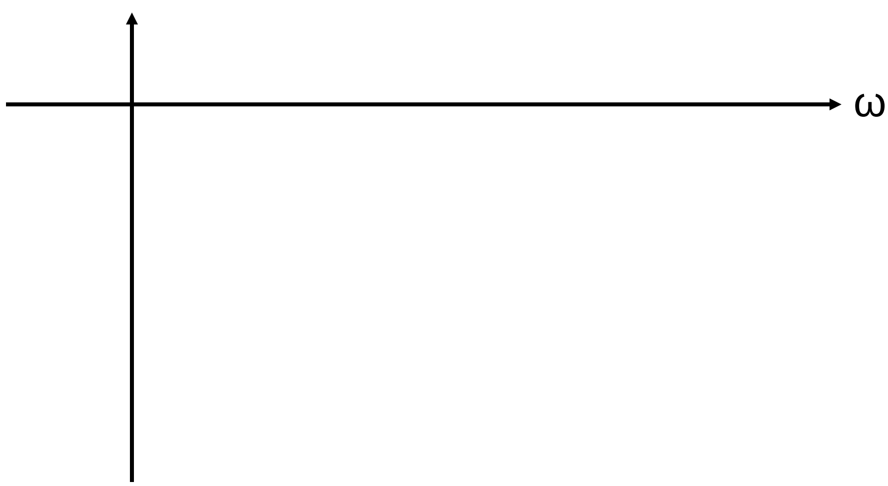
$$H(j\omega)=\frac{\left(\frac{j\omega}{\omega_{\mathbf{B}1}}\right)\left(1+\frac{j\omega}{\omega_{\mathbf{B}2}}\right)\left(1+2\zeta\frac{j\omega}{\omega_{o1}}+\left(\frac{j\omega}{\omega_{o1}}\right)^2\right)\ldots}{\left(\frac{j\omega}{\omega_{\mathbf{B}3}}\right)\left(1+\frac{j\omega}{\omega_{\mathbf{B}4}}\right)\left(1+2\zeta\frac{j\omega}{\omega_{o2}}+\left(\frac{j\omega}{\omega_{o2}}\right)^2\right)\ldots}$$

$$20\log(|H(j\omega)|)=20\log\left|\left(\frac{j\omega}{\omega_{\mathbf{B}1}}\right)\right|+20\log\left|\left(1+\frac{j\omega}{\omega_{\mathbf{B}2}}\right)\right|+20\log\left|\left(1+2\zeta\frac{j\omega}{\omega_{o1}}+\left(\frac{j\omega}{\omega_{o1}}\right)^2\right)\right|$$
$$-20\log\left|\left(\frac{j\omega}{\omega_{\mathbf{B}3}}\right)\right|-20\log\left|\left(1+\frac{j\omega}{\omega_{\mathbf{B}4}}\right)\right|-20\log\left|\left(1+2\zeta\frac{j\omega}{\omega_{o2}}+\left(\frac{j\omega}{\omega_{o2}}\right)^2\right)\right|$$

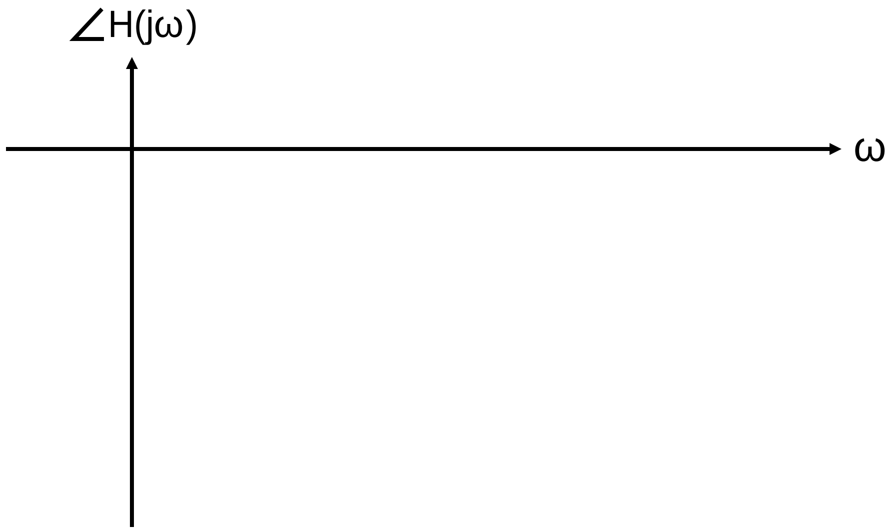
$$\angle H(j\omega)=\angle\left(\frac{j\omega}{\omega_{\mathbf{B}1}}\right)+\angle\left(1+\frac{j\omega}{\omega_{\mathbf{B}2}}\right)+\angle\left(1+2\zeta\frac{j\omega}{\omega_{o1}}+\left(\frac{j\omega}{\omega_{o1}}\right)^2\right)$$
$$-\angle\left(\frac{j\omega}{\omega_{\mathbf{B}3}}\right)-\angle\left(1+\frac{j\omega}{\omega_{\mathbf{B}4}}\right)-\angle\left(1+2\zeta\frac{j\omega}{\omega_{o2}}+\left(\frac{j\omega}{\omega_{o2}}\right)^2\right)$$

Magnitude Plot

20log(|H(jω)|)

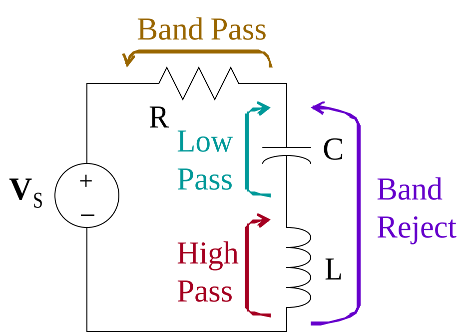


Magnitude Plot



EE40 Summer 2006: Lecture 7 Instructor: Octavian Florescu 35

More on Second-Order Filter Circuits



$\mathbf{Z} = \mathrm{R} + 1/j\omega\mathrm{C} + j\omega\mathrm{L}$

$\mathbf{H}_{BP} = \mathrm{R} / \mathbf{Z}$

$\mathbf{H}_{LP} = (1/j\omega\mathrm{C}) / \mathbf{Z}$

$\mathbf{H}_{HP} = j\omega\mathrm{L} / \mathbf{Z}$

$\mathbf{H}_{BR} = \mathbf{H}_{LP} + \mathbf{H}_{HP}$

EE40 Summer 2006: Lecture 7 Instructor: Octavian Florescu 36

Ideal Filters