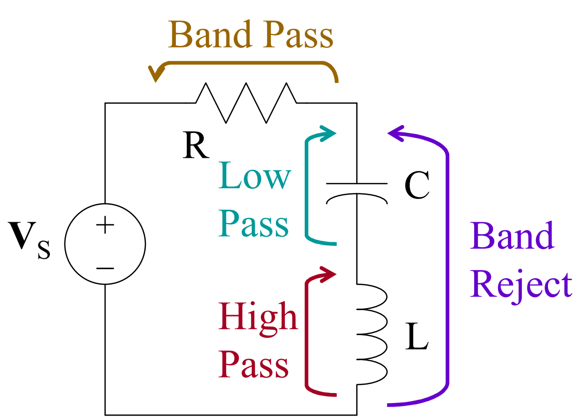
Midterm 1 Announcements

- Midterm 1:
  - 12-1:30pm on Tuesday, July 18th in Dwinelle 145.
  - Material covered in HW1-3
- HW3 due on Monday at 6pm
- Attend only your second lab slot next week
- Extended office hours
  - All day Monday: (10am-12pm and 2-5pm)

Review: Second-Order Filter Circuits



$\mathbf{Z} = R + 1/j\omega C + j\omega L$

$\mathbf{H}_{BP} = R / \mathbf{Z}$

$\mathbf{H}_{LP} = (1/j\omega C) / \mathbf{Z}$

$\mathbf{H}_{HP} = j\omega L / \mathbf{Z}$

$\mathbf{H}_{BR} = \mathbf{H}_{LP} + \mathbf{H}_{HP}$

Lecture #9

OUTLINE

- The operational amplifier ("op amp")
- Ideal op amp
- Feedback
- Unity-gain voltage follower circuit
- Summing, difference, integrator, differentiator, active filter

Reading

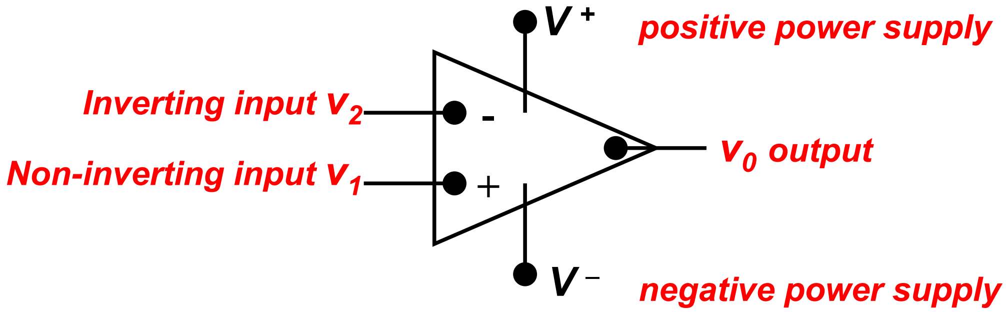
Ch. 14

The Operational Amplifier

- The ***operational amplifier*** ("***op amp***") is a basic building block used in analog circuits.
  - Its behavior is modeled using a dependent source.
  - When combined with resistors, capacitors, and inductors, it can perform various useful functions:
    - **amplification/scaling** of an input signal
    - **sign changing** (inversion) of an input signal
    - **addition** of multiple input signals
    - **subtraction** of one input signal from another
    - **integration** (over time) of an input signal
    - **differentiation** (with respect to time) of an input signal
    - **analog filtering**
    - **nonlinear functions** like exponential, log, sqrt, etc

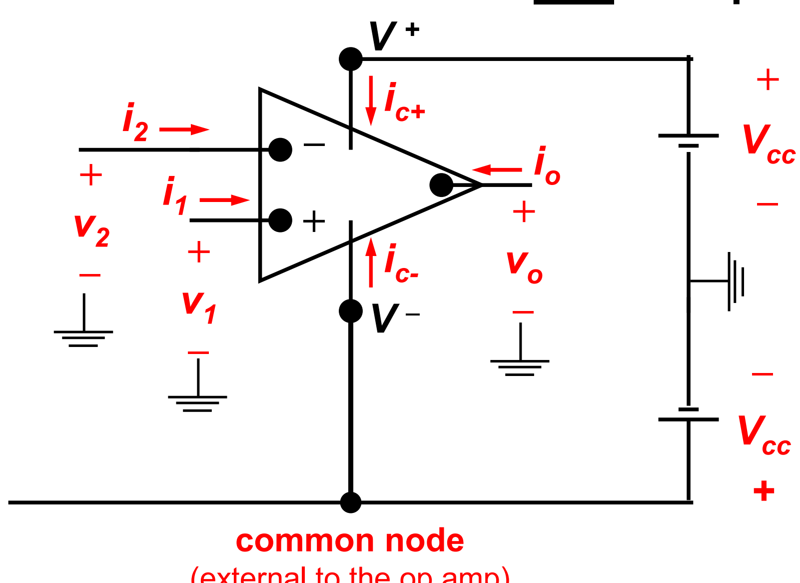
Op Amp Terminals

- 3 signal terminals: 2 inputs and 1 output
- IC op amps have 2 additional terminals for DC power supplies
- Common-mode signal= $(v_1+v_2)/2$
- Differential signal = v_1-v_2



Op Amp Terminal Voltages and Currents

- **All voltages are referenced to a common node.**
- **Current reference directions are <u>into</u> the op amp.**



Model

- A is differential gain or open loop gain
- Ideal op amp

$$A \to \infty$$
$$R_i \to \infty$$
$$R_o = 0$$

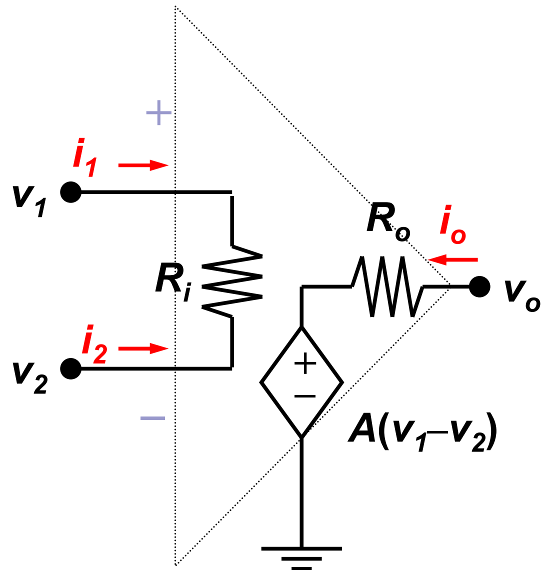
  - Common mode gain = 0

$$v_{cm} = \frac{(v_1 + v_2)}{2} \quad , v_d = v_1 - v_2$$

$$v_o = A_{cm} v_{cm} + A_d v_d$$

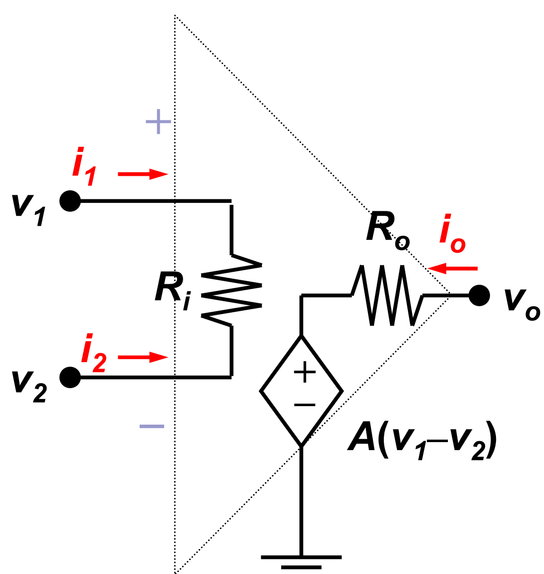
$$Since \quad v_o = A(v_1 - v_2) \quad , A_{cm} = 0$$

- Circuit Model



Model and Feedback

- Negative feedback
  - connecting the output port to the negative input (port 2)
- Positive feedback
  - connecting the output port to the positive input (port 1)



Note on negative feedback

$$V_{out} = A\varepsilon$$

$$\varepsilon = Vin - FV_{out}$$

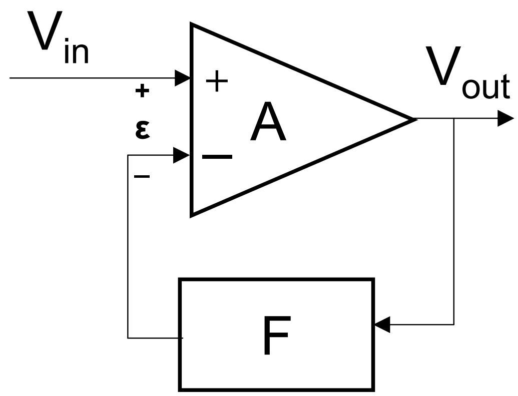
$$V_{out} = A(Vin - FV_{out})$$

$$V_{out}(1 + AF) = AV_{in}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{A}{1 + AF} \cong \frac{1}{F}$$

$$\varepsilon = \frac{V_{out}}{A}$$

$$\Rightarrow \frac{\varepsilon}{V_{in}} = \frac{1}{1 + AF} \cong 0$$



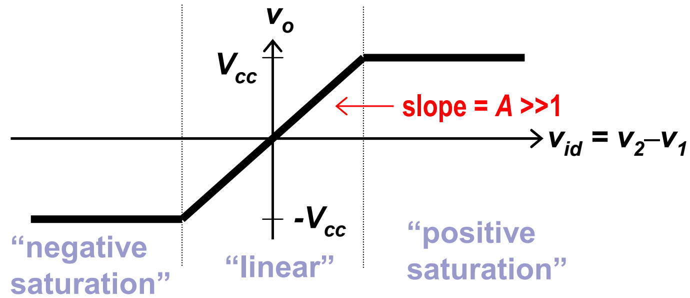
When A is very large

Summing-Point Constraint

- Check if under negative feedback
  - Small v_i result in large v_o
  - Output v_o is connected to the inverting input to reduce v_i
  - Resulting in v_i=0
- Summing-point constraint
  - $v_1 = v_2$
  - $i_1 = i_2 = 0$
- Virtual short circuit
  - Not only voltage drop is 0 (which is short circuit), input current is 0
  - This is different from short circuit, hence called "virtual" short circuit.

Op Amp Voltage Transfer Characteristic

The op amp is a differentiating amplifier:

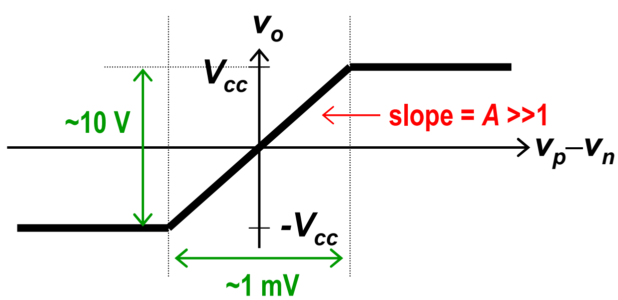


- In the linear region, $v_o = A\ (v_2 – v_1) = A\ v_{id}$
  - *A* is the open-loop gain
- Typically, $V_{cc} \le 20$ V and $A > 10^4$
  - linear range: -2 mV $\le v_{id} = (v_2 – v_1) \le 2$ mV
- Thus, for an op amp to operate in the linear region,
  - $v_2 \cong v_1$
  - There is a "virtual short" between the input terminals.)

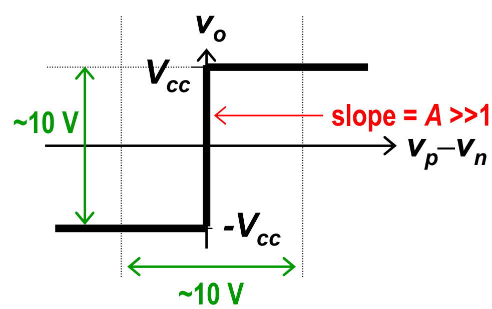
Achieving a "Virtual Short"

- Recall the voltage transfer characteristic of an op amp:

Plotted using different scales for v_o and $v_p – v_n$



Plotted using similar scales for v_o and $v_p – v_n$

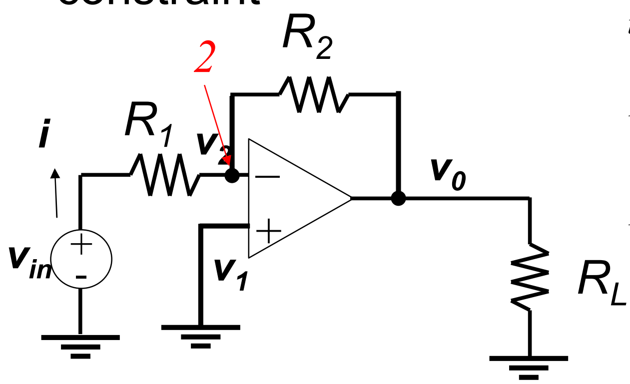


<u>Q</u>: How does a circuit maintain a virtual short at the input of an op amp, to ensure operation in the linear region?

<u>A</u>: By using ***negative feedback***. A signal is fed back from the output to the <u>inverting</u> input terminal, effecting a ***stable*** circuit connection. Operation in the ***linear region*** enforces the virtual short circuit.

Inverting Amplifier

- Negative feedback → checked
- Use summing-point constraint



$$Closed\ \ loop\ \ gain = A_v = \frac{v_o}{v_{in}}$$

$$v_1 = v_2 = 0\ \ ,i_1 = i_2 = 0$$

$$Use\ \ KCL\ \ At\ \ Node\ \ 2.$$

$$i = \frac{(v_{in} - v_2)}{R_1} = \frac{(v_{out} - v_2)}{R_2}$$

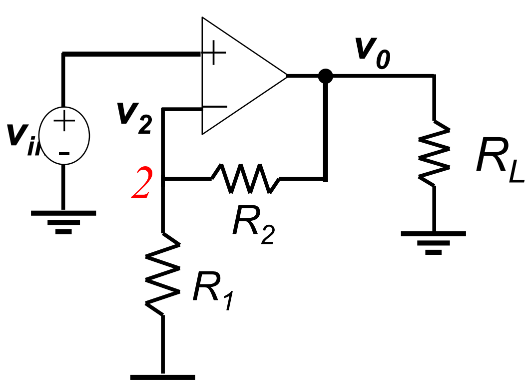
$$v_o = -\frac{R_2 v_o}{R_1}$$

$$Input\ \ impedance = \frac{v_{in}}{i} = R_1$$

Ideal voltage source – independent of load resistor

Non-Inverting Amplifier

- Ideal voltage amplifier



$$Closed\ \ loop\ \ gain = A_v = \frac{v_o}{v_{in}}$$

$$v_1 = v_2 = v_{in}\ \ ,i_1 = i_2 = 0$$

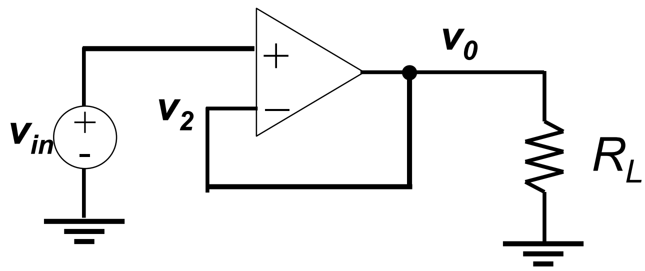
$$Use\ \ KCL\ \ At\ \ Node\ \ 2.$$

$$i = \frac{(v_0 - v_2)}{R_2} = \frac{(v_2 - 0)}{R_1}$$

$$A = \frac{v_o}{v_{in}} = \frac{(R_1 + R_2)}{R_1}$$

$$Input\ \ impedance = \frac{v_{in}}{i} \to \infty$$

Voltage Follower



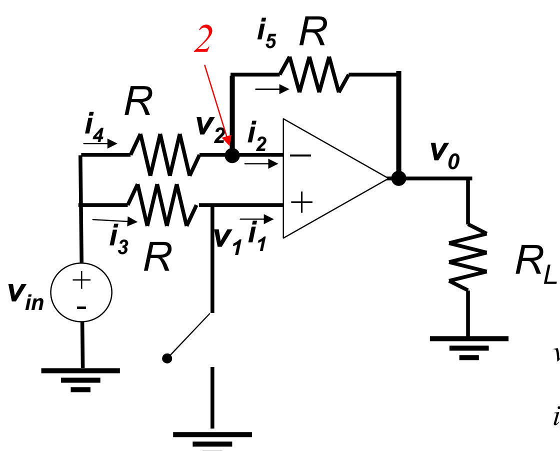
$$R_2 = 0$$

$$R_1 \to \infty$$

$$i = \frac{(v_0 - v_2)}{R_2} = \frac{(v_2 - 0)}{R_1}$$

$$A = \frac{v_o}{v_{in}} = \frac{(R_1 + R_2)}{R_1} = 1 + \frac{R_2}{R_1} = 1$$

Example 1



- Switch is open

$$v_1 = v_2 \quad , i_1 = 0 \to i_3 = 0$$

$$i_3 = \frac{(v_{in} - v_1)}{R} \to v_1 = v_2 = v_{in} \to i_4 = 0 \to i_5 = 0$$

$$i_5 = \frac{(v_0 - v_2)}{R} \to v_0 = v_2 = v_{in}$$

$$A = \frac{v_o}{v_{in}} = 1 \quad , R_{in} \to \infty$$

- Switch is closed

$$v_1 = v_2 = 0 \quad , i_1 = 0 \to i_3 = 0$$

$$i_4 = \frac{(v_{in} - v_2)}{R} = i_5 = -\frac{(v_0 - v_2)}{R}$$

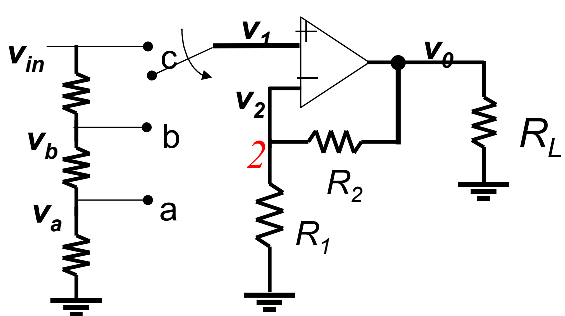
$$v_0 = -v_{in}$$

$$A = \frac{v_o}{v_{in}} = -1 \quad , R_{in} = R/2$$

Example 2

- Design a variable gain amplifier
  - Must have 3 gain settings that can be selected by switches
    - A = 10, 1, 0.1
  - The input resistance is 1MΩ

Example 2



$$v_o = (1 + \frac{R_2}{R_1}) v_1$$

$$v_1 = v_{in} \quad Switch \;\; at \;\; c$$

$$v_1 = \frac{R_a + R_b}{R_a + R_b + R_c} v_{in} \quad Switch \;\; at \;\; b$$

$$v_1 = \frac{R_a}{R_a + R_b + R_c} v_{in} \quad Switch \;\; at \;\; a$$

Example 2 (cont'd)

$$R_{in} = R_a + R_b + R_c = 1M\Omega$$

$$Max\ \ A_v = 10 = (1 + \frac{R_2}{R_1})\ \ \ Switch\ \ at\ \ c$$

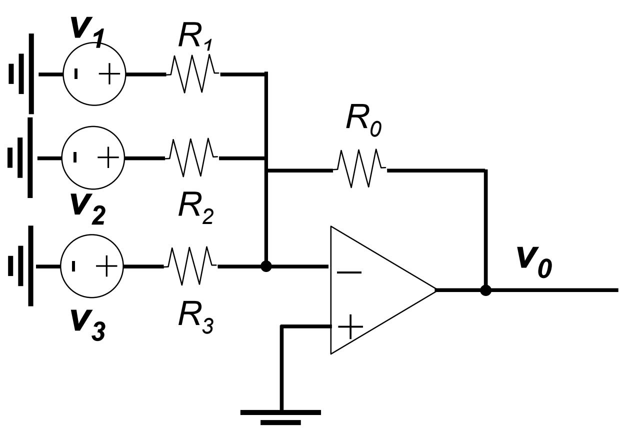
$$A_v = 1 = \frac{R_a + R_b}{R_a + R_b + R_c}(1 + \frac{R_2}{R_1})\ \ \ Switch\ \ at\ \ b \therefore \frac{R_a + R_b}{R_a + R_b + R_c} = 0.1$$

$$A_v = 0.1 = \frac{R_a}{R_a + R_b + R_c}(1 + \frac{R_2}{R_1})\ \ \ Switch\ \ at\ \ a \therefore \frac{R_a}{R_a + R_b + R_c} = 0.01$$

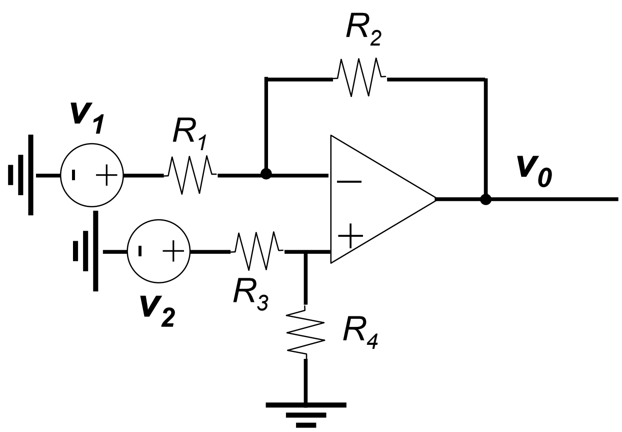
$$\therefore R_a = 10k\Omega, R_b = 90k\Omega, R_c = 900k\Omega$$

$$R_2 = 9R_1$$

Summing Amplifier

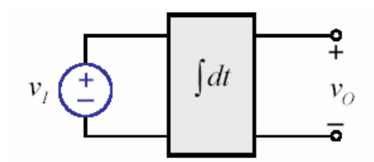


Difference Amplifier

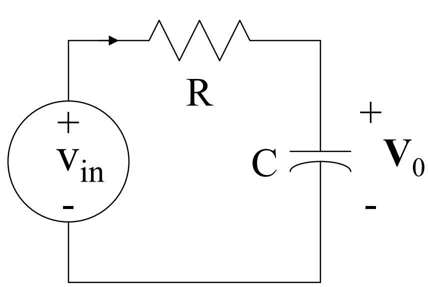


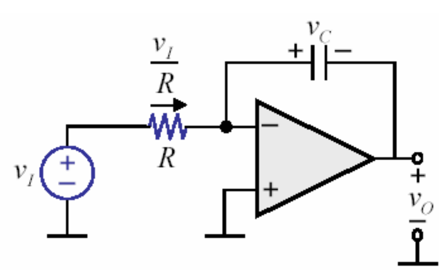
Integrator

- Want $v_o = K\int v_{in}dt$



- What is the difference between:



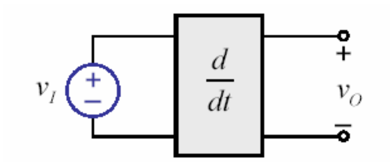


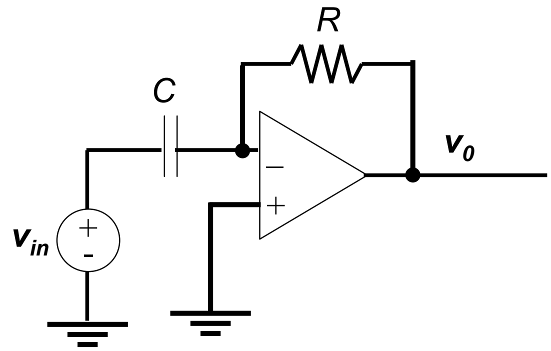
$$v_O \approx \frac{1}{RC}\int_{-\infty}^{t} v_I \, dt$$

$$v_O = -\frac{1}{C}\int_{-\infty}^{t} \frac{v_I}{R} dt$$

Differentiator

- Want

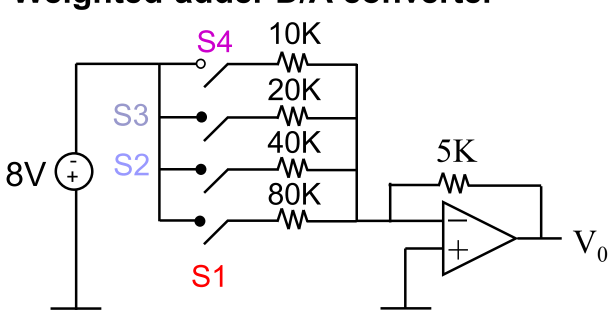




Application: Digital-to-Analog Conversion

A DAC can be used to convert the digital representation of an audio signal into an analog voltage that is then used to drive speakers -- so that you can hear it!

“Weighted-adder D/A converter”



4-Bit D/A

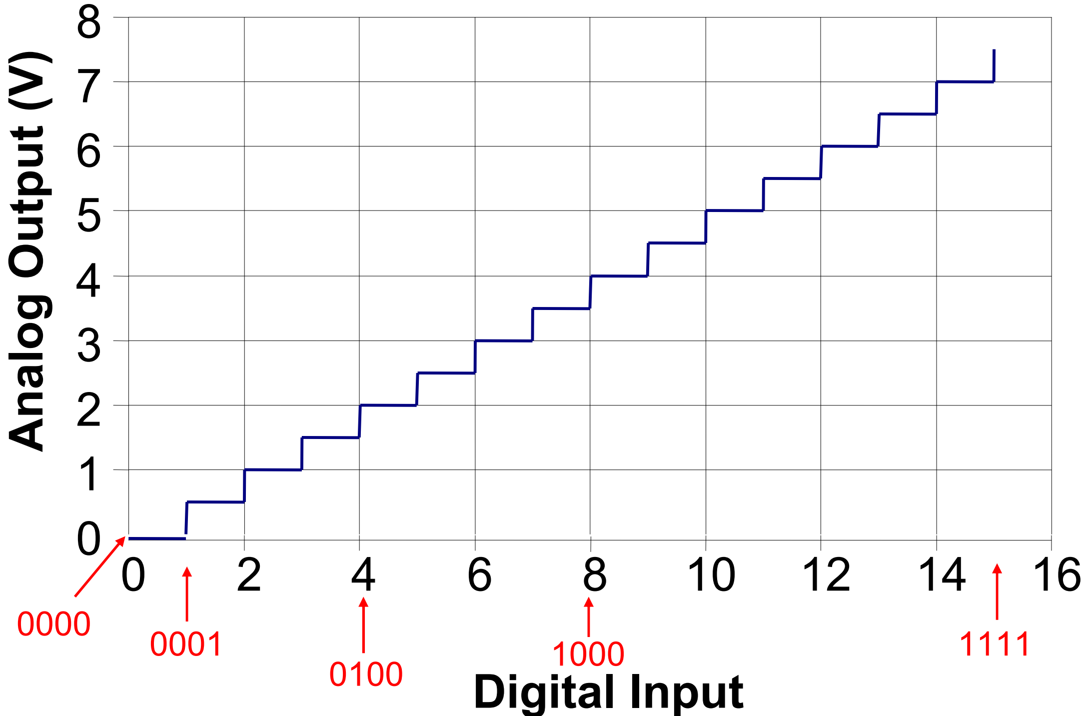
S1 closed if LSB =1
S2 " if next bit = 1
S3 " if " " = 1
S4 " if MSB = 1

(Transistors are used as electronic switches)

Binary number	Analog output (volts)
0 0 0 0	0
0 0 0 1	.5
0 0 1 0	1
0 0 1 1	1.5
0 1 0 0	2
0 1 0 1	2.5
0 1 1 0	3
0 1 1 1	3.5
1 0 0 0	4
1 0 0 1	4.5
1 0 1 0	5
1 0 1 1	5.5
1 1 0 0	6
1 1 0 1	6.5
1 1 1 0	7
1 1 1 1	7.5

↑ MSB ↑ LSB

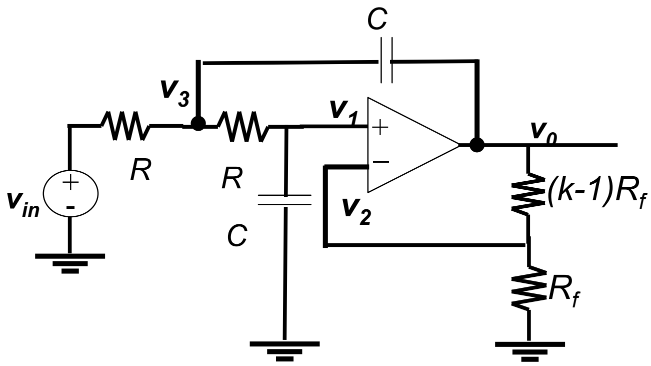
Characteristic of 4-Bit DAC



Active Filter

- Contain few components
- Transfer function that is insensitive to component tolerance
- Easily adjusted
- Require a small spread of components values
- Allow a wide range of useful transfer functions

Active Filter Example



$$v_1 = v_2 = \frac{v_o}{k}$$

$$Use\ \ KCL\ \ At\ \ Node\ \ A \Rightarrow \frac{(v_3 - v_1)}{R} = j\omega C v_1$$

$$Use\ \ KCL\ \ At\ \ Node\ \ B \Rightarrow \frac{(v_{in} - v_3)}{R} = j\omega C(v_3 - v_o) + \frac{(v_3 - v_1)}{R}$$

$$\frac{v_o}{v_{in}} = \frac{k}{1 - \omega^2 R^2 C^2 + j\omega RC(3-k)}$$

$$Let\ \omega_B = 1/RC$$

$$|H(\omega)| = \left|\frac{v_o}{v_{in}}\right| = \frac{k}{\sqrt{\left(1 - \frac{\omega^2}{{\omega_B}^2}\right)^2 + \frac{\omega^2}{{\omega_B}^2}(3-k)^2}}$$

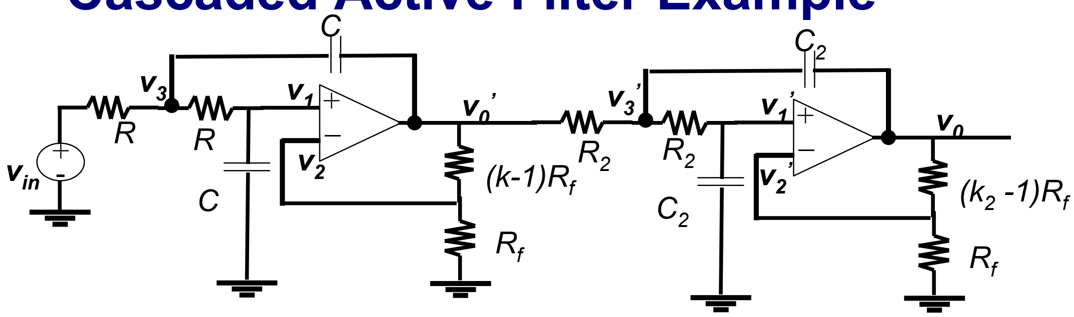
$$\omega = 0, |H(\omega)| = k\ \ DC\ \ gain$$

$$\omega = \omega_B, |H(\omega)| = \frac{k}{3-k}$$

$$\omega >> \omega_B, |H(\omega)| = \frac{k}{\left(\frac{\omega^2}{{\omega_B}^2}\right)} \propto \omega^{-2}$$

$$20\log|H(\omega)|\ \ decays\ at\ a\ rate\ of\ 40dB/decade$$

Cascaded Active Filter Example



$$\frac{v_o}{v_{in}} = \frac{k_2}{1 - \omega^2 R_2^2 C_2^2 + j\omega R_2 C_2 (3-k_2)} \frac{k}{1 - \omega^2 R^2 C^2 + j\omega RC(3-k)}$$

$$Let\ \omega_B = 1/RC,\ \ \omega_{B2} = 1/R_2 C_2$$

$$|H(\omega)| = \left|\frac{v_o}{v_{in}}\right| = \frac{k_2}{\sqrt{\left(1 - \frac{\omega^2}{{\omega_{B2}}^2}\right)^2 + \frac{\omega^2}{{\omega_{B2}}^2}(3-k_2)^2}} \frac{k}{\sqrt{\left(1 - \frac{\omega^2}{{\omega_B}^2}\right)^2 + \frac{\omega^2}{{\omega_B}^2}(3-k)^2}}$$

$$\omega = 0, |H(\omega)| = k_2 k\ \ DC\ \ gain$$

$$\omega = \omega_B, |H(\omega)| = \frac{k_2}{3-k_2}\frac{k}{3-k}$$

$$\omega >> \omega_B, |H(\omega)| = \frac{k_2 k}{\left(\frac{\omega^4}{{\omega_{B2}}^2 {\omega_B}^2}\right)} \propto \omega^{-4}$$

$$20\log|H(\omega)|\ \ decays\ at\ a\ rate\ of\ 80dB/decade$$