Lab 10 Sampling and Frequency Aliasing

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EE145M

Lab Time: 9-12pm Wednesday
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Station 8

Aim

To analyst frequency aliasing and to sample sine waves of various frequencies and output the sampled data to the computer through the FPGA analog input and analog output ports. The FPGA replace the external A/D or D/A converters.

1. Setup

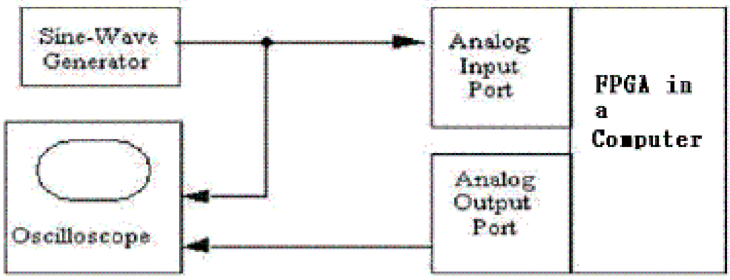


Figure 1.1. Setup Diagram.

The FPGA is equipped with differential inputs. In order to use the differential inputs as a single ended input, feed the signal from the signal generator into AIO1+, and connect AI0- to the signal generator ground. AIO1- should also be shorted to AIGND1.

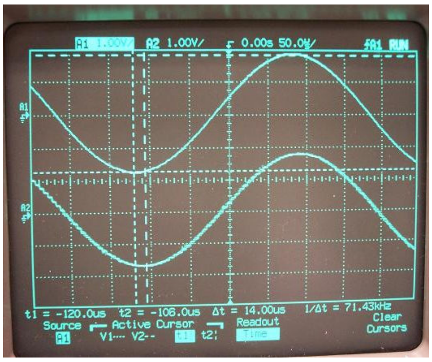


Figure 1.2. An input waveform of 2.5kHz is being sampled by at 10kHz.

2. Data summary

2.1 From the 0.1 fs sine wave in procedure section 2, determine your sampling frequency in Hz. Frequency aliasing will not occur with a slow sine wave.

Instead of finding the sampling frequency that gives exactly 10 samples per cycle as seen on the oscilloscope, a 10 kHz sampling frequency is used instead according to the lab manual. Aliasing did not occur with an input waveform of the frequency 1 kHz.

2.2 For all the sine-waves and triangle-waves you sampled, prepare a table that compares the actual input frequency f with the apparent frequency f0 of the sampled waveform. Include a column of the expected apparent frequency f0 derived from Chapter 3, Figure 3.32.

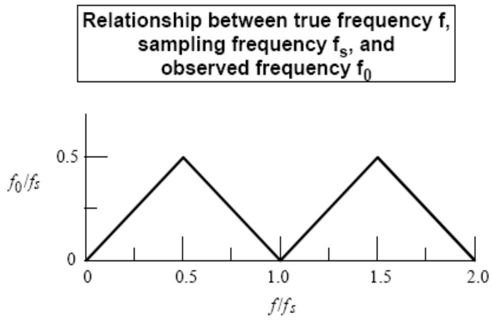


Figure 2.2 [Derenzo, 185]

According to TA Peter Lau, Figure 3.33 is used instead of Figure 3.32 to generate the column of the expected apparent frequency. The apparent frequency is determined by observing the waveform, and counting the number of sample points within one cycle. For example, if there are 4 data points within a period, the frequency is 10kHz/4=2.5kHz. The sampling frequency (10kHz) is four times faster than the input frequency (2.5kHz), so there are four data points per cycle.

Actual Input Frequency (kHz)	Apparent Frequency fo (kHz)	Expected Apparent Frequency (kHz)
Sine Wave		
1	1	1
2.5	2.5	2.5
5	5	5
7.5	2.5	2.5
9.9	0.1	0.1
15	5	5
17.5	2.5	2.5
19.9	0.1	0.1
Triangular Wave		
1	1	1
2.5	2.5	2.5
5	5	5
7.5	2.5	2.5
9.9	0.1	0.1

3. Discussion

3.1 Describe and discuss your observations from procedure sections 2 and 3. Explain how you measured the sampling frequency in procedure section 2. Discuss the waveforms you observed in procedure sections 2 and 3. Pay particular attention to the detailed shape of the waveforms, phase shift between the waveforms, and relative p-p amplitudes.

The Nyquist rate is half of the sampling frequency. In this case, the Nyquist rate is 5 kHz. The shape of the waveform appears to be discrete points. This is because sampling does not happen continuously, it occurs at discrete time. Therefore, for a continuous sine wave, the sampled data will be discrete data points with certain amplitudes and certain distance apart.
If the input waveform has a frequency lower than the Nyquist rate, which is lower than 5 kHz in this case, then the output waveform accurately reproduces the input waveform. In other words, the relative peak-to-peak amplitudes of the input waveform and the output waveform are the same. If the input waveform has a frequency higher than the Nyquist rate but lower than the sampling frequency (10kHz), then aliasing occurs. The apparent output frequency is lower than the actual input frequency, and the peak-to-peak amplitude decreases as input frequency increases. As the input frequency increases within this range, the output frequency decreases from the Nyquist rate to 0Hz. The relationship of observed frequency and sampling frequency is shown in Figure 3.1.

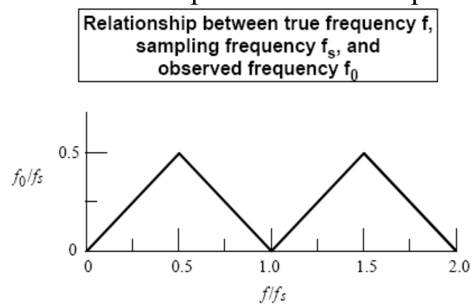
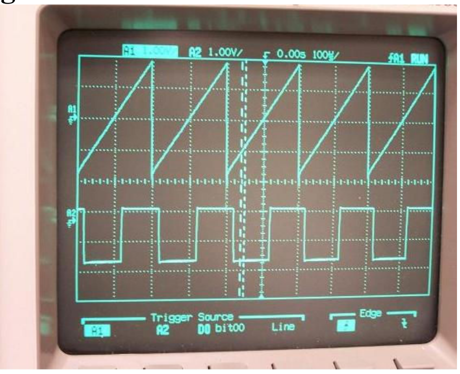


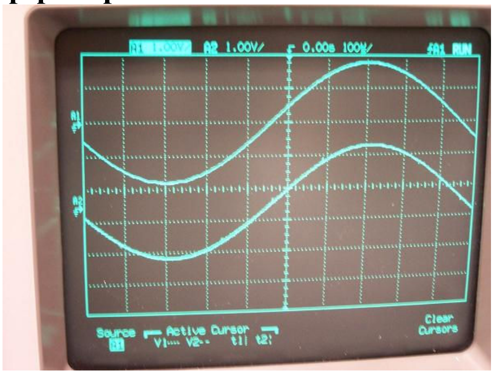
Figure 3.1[Derenzo, 185]

If the input frequency is above the sampling frequency, then the waveform apparent frequency will resemble that of a lower frequency. This relationship of input and apparent frequency is also shown in Figure 3.1.

There is no phase shift between the waveforms. This is because the FPGA used in the lab has exact frequency. In old machines, if there is a delay caused by software during sampling, then there will be phase shift between the input and output waveforms.

3.2 Draw conclusions about what you have learned about sampling. For example, if you are only able to sample a few cycles of a pure sine wave, how frequently do you need to sample to get an accurate measurement of its p-p amplitude?





Triangular Input 5kHz Input 5kHz

If the input sine wave has a frequency at or below half of the sampling frequency, accurate measurements of the peak-to-peak amplitude can be measured. If the input sine wave has a frequency higher than half of the sampling frequency, the peak-to-peak amplitude appears to be lower than the input amplitudes. As the input frequency go further beyond half of the input frequency, the sampled waveform appears to have a lower frequency and lower peak-to-peak amplitude. For example, if I am only able to sample a few cycles of a pure sine wave, I need to sample at least twice as fast as the input waveform to get an accurate measurement of its peak-to-peak amplitude.

3.3 Discuss the ability and limitations of this technique to sample analog waveforms, store them digitally, and recover them at a later time. (*Note*: the audio compact disc is an important application of these techniques.)

The ability of sampling is to be able to take an analog waveform, and store the information of the analog waveform digitally on the computer. The information in the computer can be taken out at a later time to reproduce the same analog waveform, for example, playing the audio compact disc is an example of this ability.

The limitation of sampling is the input waveform cannot have a higher frequency than half of the sampling frequency or aliasing will occur. For input waveform of higher frequency, the sampling frequency needs to be twice as fast. This puts an upper limit on how fast an input waveform can be accurately sampled with accurate frequency and peak-to-peak amplitude.

4. Questions

4.1 What was your sampling frequency in procedure section 3?
The sampling frequency is 10 kHz in procedure section 3.

4.2 Were the apparent frequencies of the recovered analog waveforms approximately the same for sine-wave input frequencies of 0.25 f_s and 0.75 f_s? Explain.

Yes, the sine-wave input frequencies of 0.25fs (2.5kHz) and 0.75fs (0.75kHz) appears to be the same. This is because the 0.75fs frequency exceeds the Nyquist frequency (0.5fs), so the waveform appears to be at a lower frequency. The output frequency will appear to be lower than half of the Nyquist frequency. In this case, the 0.75fs waveform appears to be exactly 0.25fs. The relationship between the apparent frequency fo and the input frequency f is shown in Figure 4.2.

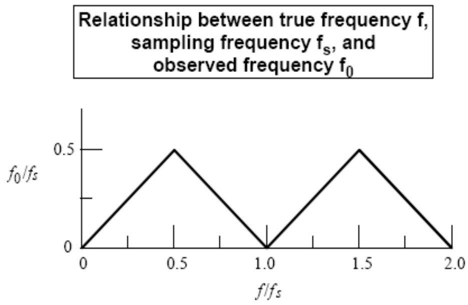


Figure 4.2 [Derenzo, 185]

4.3 What is the maximum frequency sine wave that your system can sample without aliasing?

The maximum frequency sine wave the system can sample is half of the sampling frequency, which is 0.5*10kHz=5kHz.

4.4 Did aliasing at $f \approx fs$ change the shape of the triangle wave? Explain your answer in terms of progressive shifts in the sampling time?

No, when f ≈fs the aliasing did not change the shape of the triangular wave. This is because the input waveform is periodic, and every time the waveform was sampled at a slightly different point of the periodic wave. The situation is like having an array of identical books lining up, and you take the first page from the first book, the second page from the second book, the third page from the third book, and so on. By doing this, the fs can measure input waveforms of close to sampling frequency, and still be able to display the shape of the input waveform. In fact, oscilloscopes use techniques similar to having $f \approx fs$ to display input waveforms of higher frequency.

4.5 How would you sample a periodic 1 MHz waveform so that the sampled values had the same shape as the original waveform but occurred at a slower frequency of 1 kHz?

I would set the sampling frequency to 0.999 kHz, which will produce an aliasing effect. I can take advantage of the aliasing effect by taking a slightly different point of a periodic wave, which will give me the same shape as the original waveform. This is a similar technique of "taking the first page of the first book, second page of the second book…" technique.

4.6 What is the maximum sample rate of the setup? Bear in mind that in each iteration, two conversions occur in series, first A/D, then D/A. Look up the A/D and D/A conversion rates in the datasheet for the PCI-7831R FPGA – which creates the dominating overhead? Does your maximum sample rate seem reasonable?

The FPGA has a minimum sampling period of 212 ticks. 1 tick equals 1/40MHz = 25us. The maximum sampling frequency is 1/(25us per tick*212 ticks)=188.7Hz.

Settling Time

Step Size	Accuracy		
	16 LSB	4 LSB	2 LSB
±20.0 V	7.5 μs	10.3 μs	40 μs
±2.0 V	2.7 μs	4.1 μs	5.1 μs
±0.2 V	1.7 μs	2.9 μs	3.6 μs

[National Instrument datasheet]

Table B-2. NI 783xR I/O Signal Summary

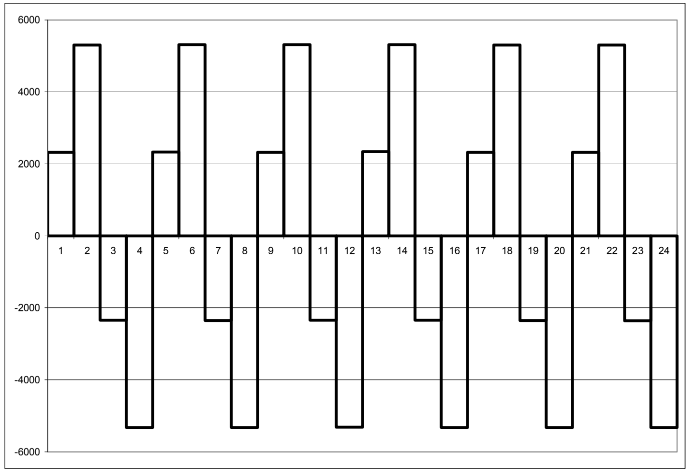
Signal Name	Signal Type and Direction	Impedance Input/ Output	Protection (Volts) On/Off	Source (mA at V)	Sink (mA at V)	Rise Time	Bias
+5V	DO	—	—	—	—	—	—
AI<0..7>+	AI	10 GΩ in parallel with 100 pF	42/35	—	—	—	±2 nA
AI<0..7>–	AI	10 GΩ in parallel with 100 pF	42/35	—	—	—	±2 nA
AIGND	AO	—	—	—	—	—	—
AISENSE	AI	10 GΩ in parallel with 100 pF	42/35	—	—	—	±2 nA
AO<0..7>	AO	1.25 Ω	Short circuit to ground	2.5 at 10	2.5 at –10	10 V/μs	—
AOGND	AO	—	—	—	—	—	—
DGND	DO	—	—	—	—	—	—
DIO<0..15> Connector 0 DIO<0..39> Connector <1..2>	DIO	—	–0.5 to +7.0	5.0 at 2.4	5.0 at 0.4	12 ns	—
AI = Analog Input AO = Analog Output DIO = Digital Input/Output DO = Digital Output							

The A/D conversion is in the order of microseconds, while D/A conversion is in the order of nano-seconds. A/D conversion clearly takes longer to convert. The measured minimum sampling period is 25us for a 2V peak-to-peak input signal. According to the datasheet, the conversion should take less than 25us. There may be other delays that keep the sampling time from going below 25us.

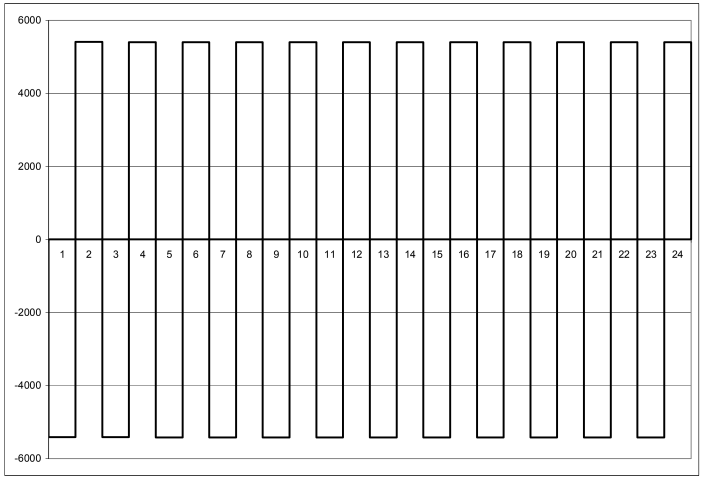
5. Laboratory Data Sheets

Printouts for Sine Wave Waveforms

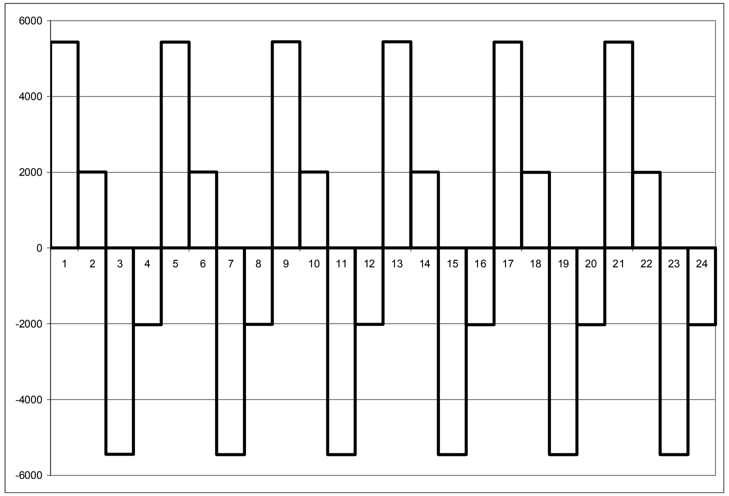
3.1 0.25fs = 2.5kHz



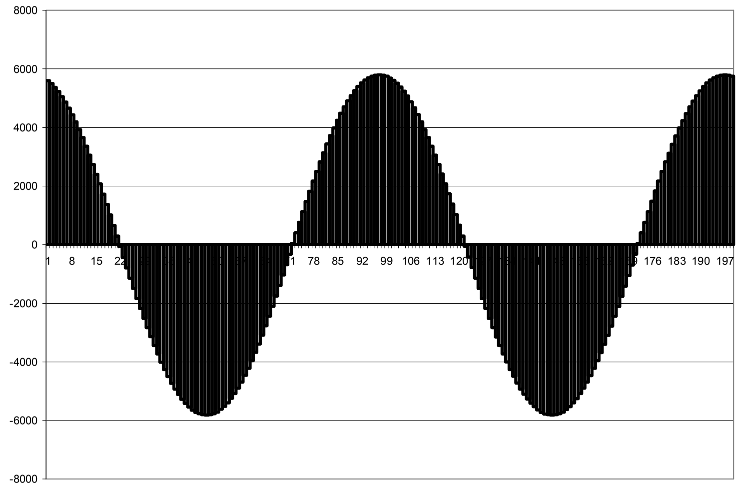
3.2 0.5fs = 5kHz



3.3 0.75fs = 7.5kHz

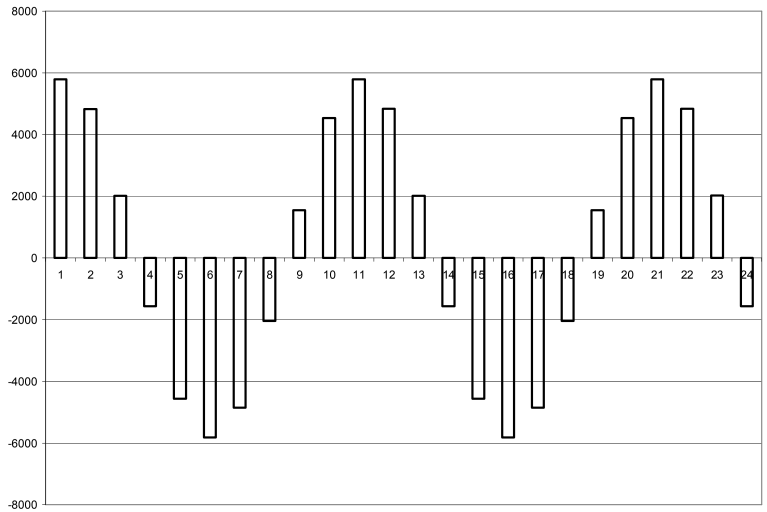


3.4 0.99fs = 9.9kHz

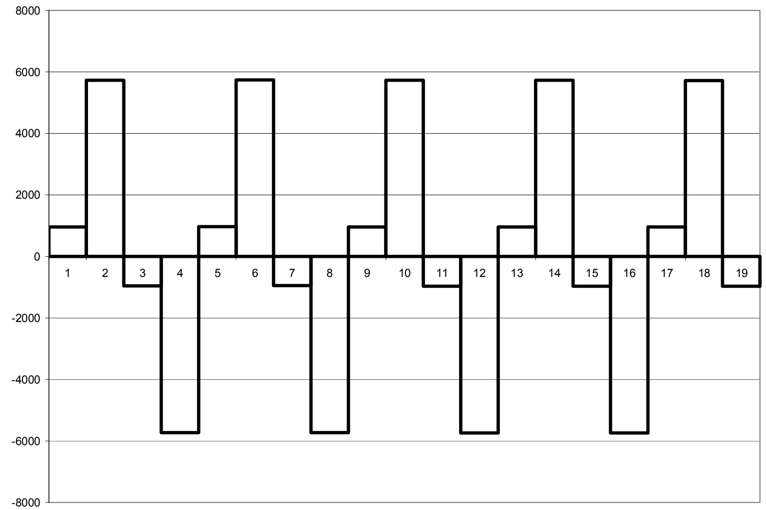


Printout for 1.5fs, 1.75fs, and 1.99fs

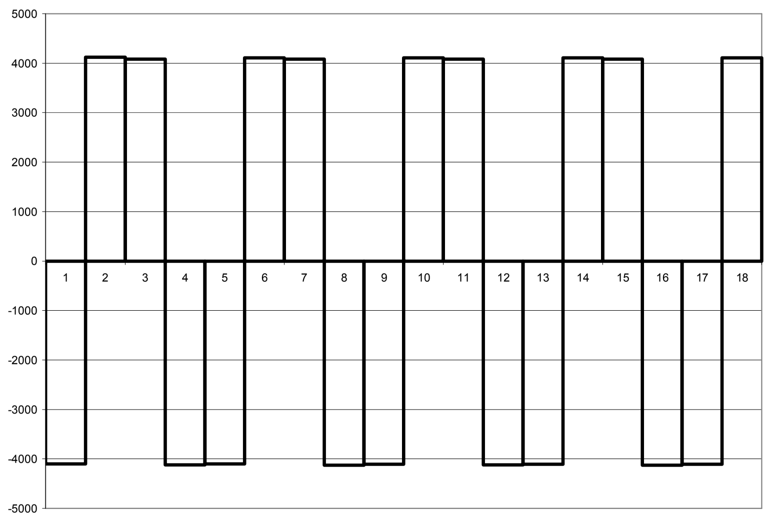
2.1 0.1fs = 1kHz



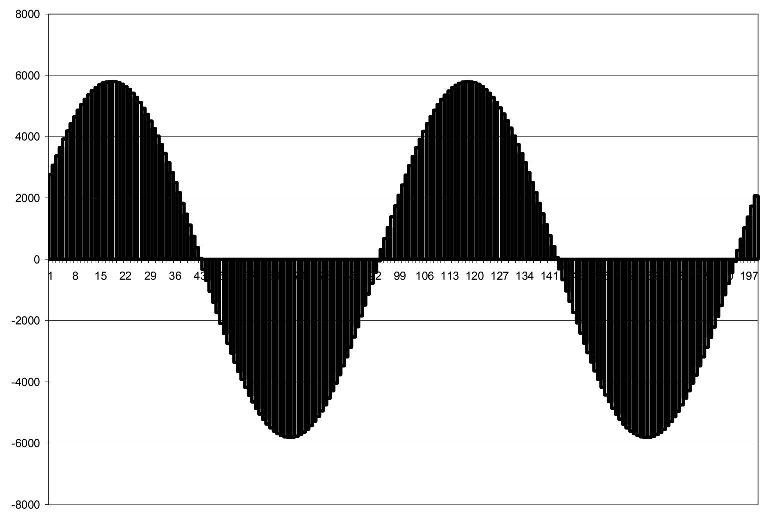
3.5 1.5fs = 15kHz



3.5 1.75fs = 17.5kHz

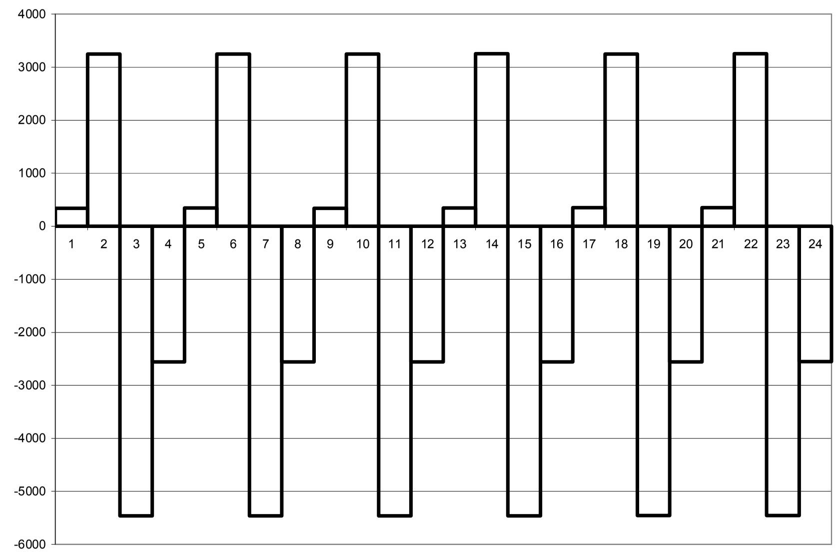


3.5 1.99fs = 19.9kHz

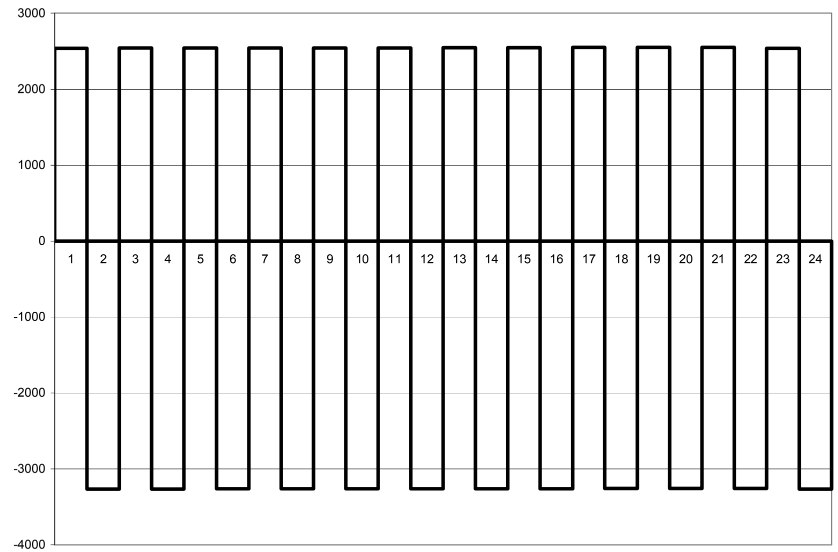


Printouts for Triangular Wave Waveforms

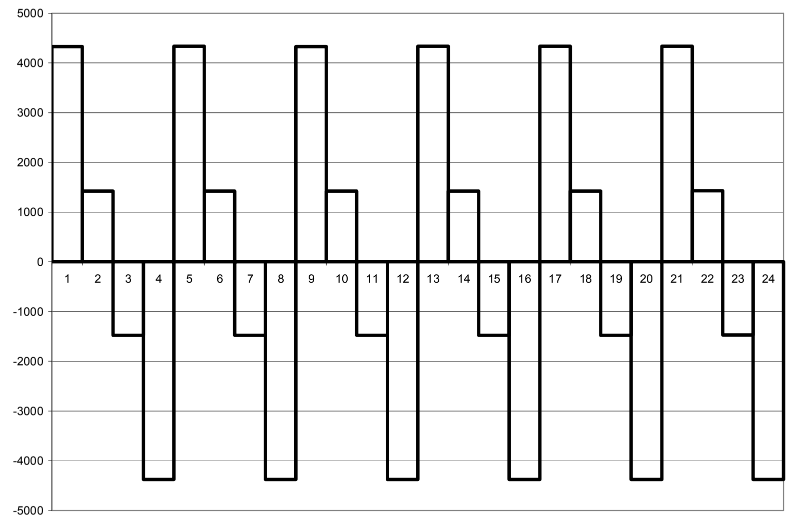
4.1 0.25fs = 2.5kHz



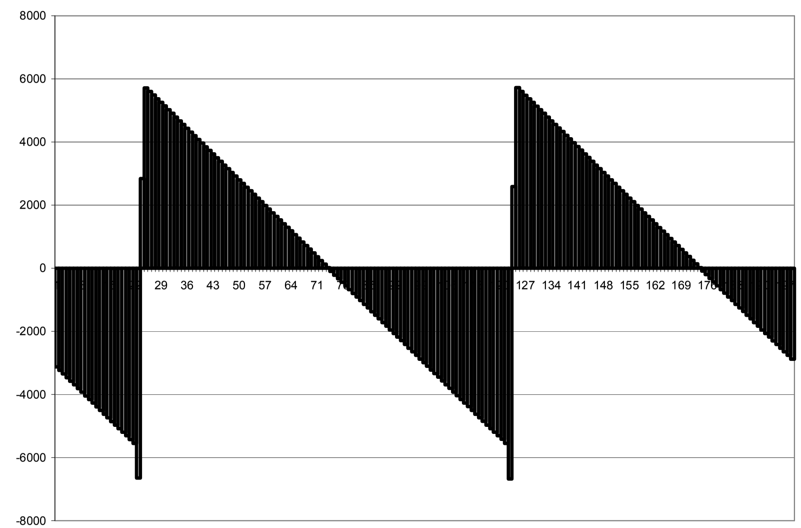
4.2 0.5fs = 5kHz



4.3 0.75fs = 7.5kHz

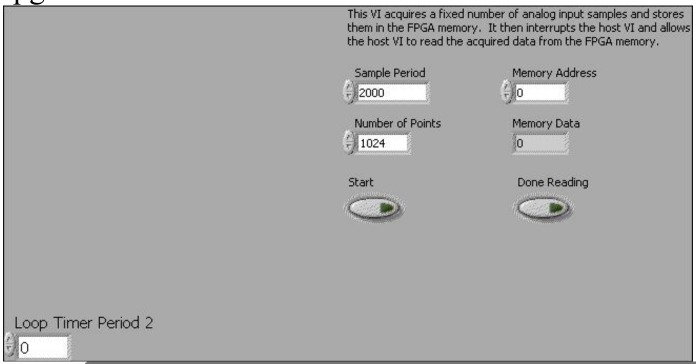


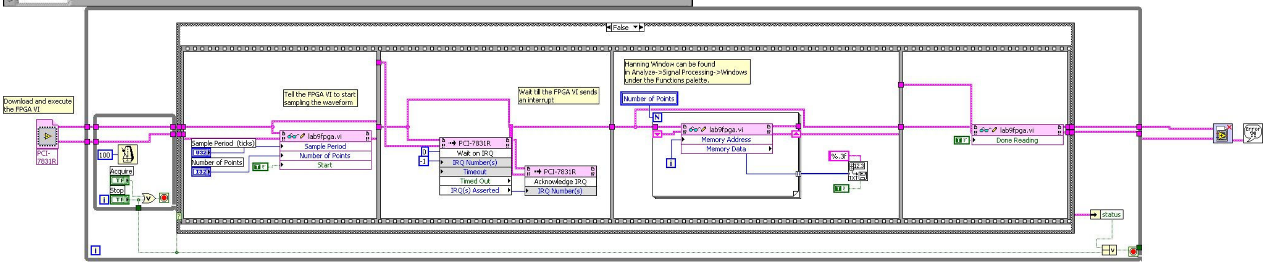
4.4 0.99fs = 9.9kHz



LabView VI

fpga.vi





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